Optimal Allocation of Social Cost for Electronic Payment System: A Ramsey Approach

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Using a standard Ramsey approach, we examine the optimal allocation of social cost for an electronic payment system in the context of a dynamic general equilibrium model. The benevolent government provides electronic payment services and allocates relevant social cost through taxation on the beneficiaries' labor and consumption. A higher tax rate on labor yields the following desirable allocations. First, it implies a lower welfare loss because of the distortionary consumption taxation. It also enhances the economy of scale in the use of electronic payment technology, reducing per transaction cost of electronic payment. Finally, it saves the cost of withdrawing and carrying around cash by reducing the frequency of cash trades. All these channels together imply optimality of the unity tax rate on labor.

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I. Introduction

An electronic payment system requires record-keeping and information-processing technologies for transaction clearing and verification purposes. However, from the society's point of view, the installment and operation of such technologies incur some resource cost. Hayashi and Keeton (2012) refer to this resource cost as the "social cost" of using electronic payment methods and distinguish it from "private cost" that includes the fees imposed by one party to another. This social cost has the distinct feature of being composed largely of fixed costs that do not rely on the frequency or value of the transactions. Once the electronic record-keeping and information-processing devices are installed at a substantial cost, electronic payment services could be provided with trivial marginal cost.

Considering the high social cost incurred in the installment of electronic payment technology, it is quite natural to ask how the social cost should be allocated. However, to the best of our knowledge, no study has thoroughly addressed this question. Most recent studies on electronic payment systems have focused on issues related to private cost (*e.g.*, Prager *et al.* 2009; Rochet and Wright 2010; Verdier 2011; Lee 2014) or on issues related to the competition among payment instruments (*e.g.*, Li 2011; Kim and Lee 2012; Lotz and Vasseli 2013; Telyukova 2013).

This paper examines an optimal allocation of the social cost for an electronic payment system by incorporating a standard Ramsey approach into a dynamic general equilibrium model where money is essential. Specifically, we adopt a standard search-theoretic monetary model augmented with distribution of wealth. We consider wealth heterogeneity across agents to capture economy of scale in the use of an electronic payment system. At the beginning of a period, each agent becomes a buyer, a seller, or a non-trader with some probability. Each buyer is then relocated randomly to match with a seller. After the realization of such relocation shock, a buyer determines whether to carry cash or to open a checking account for subsequent pairwise trade. Carrying cash incurs disutility cost because of the inconvenience and the risk of loss. A buyer can also pay in cash by withdrawing the amount from her checking account where, for simplification, cash-withdrawing cost is assumed to be the same as its carrying cost. A buyer opening a checking account can also use an electronic payment system by shouldering its fixed installment cost.

As agents cannot commit to their future actions and trading histories are private, all trades in a pairwise meeting are *quid pro quo*. Hence, either cash or checking-account deposit should be transferred in exchange for goods produced where the terms of trade are determined by a buyer's take-it-or-leave-it offer. Electronic transactions based on checking accounts require the installment of an electronic payment system, which incurs a fixed cost to its service provider (hereinafter referred to as "the government"). The government collects the fixed cost from people who transfer and receive money via an electronic payment system. Notably, the per transaction cost of electronic payment declines as more transactions are made via the system. This captures the economy of scale in the use of electronic payment technology. As a key source of the efficiency of electronic payment, this economy of scale is determined by the choice of means of payment of heterogeneous agents with different monetary wealth.

We use the model to examine an optimal allocation of the social cost for an electronic payment system. Our Ramsey problem is a standard one; to maximize social welfare, the government chooses a policy on the allocation of the social cost for an electronic payment system across buyers and sellers who use the system in pairwise trades. Under the assumed buyer-take-all trading protocol, this problem is transformed into that of choosing a tax scheme on the beneficiaries (*i.e.*, buyers) of electronic transactions in the form of taxation on labor or consumption. As the endogeneity of a nondegenerate wealth distribution rules out closed-form solutions, the Ramsey problem is solved numerically. The main results are as follows.

The choice of taxation on a buyer's labor or consumption not only has an intensive-margin effect on the terms of pairwise trade for a given cost per electronic transaction but also an extensive-margin effect on the choice of means of payment. The welfare implications of these two effects depend crucially on a nondegenerate distribution of wealth and economy of scale in the use of an electronic payment system.

With regard to the intensive-margin effect, labor taxation turns out to have a lump-sum feature in the sense that it does not affect quantity consumed in exchange for money transferred electronically, whereas consumption taxation turns out to be distortionary in the sense that it affects quantity consumed in electronic transactions. Hence, labor taxation favors relatively poor agents whose consumption is sufficiently small, whereas consumption taxation favors relatively rich agents whose consumption is sufficiently large. As the tax rate on labor increases (or the tax rate on consumption decreases), the difference between the average quantity of good consumed by buyers and that produced by sellers decreases.

Labor taxation also has an extensive-margin effect on the choice between electronic and cash payment, which then affects the cost per electronic transaction via an economy-of-scale channel. In particular, the frequency of electronic transactions is shown to increase with the tax rate on labor. When the tax rate on labor is zero for a given cost per electronic transaction, relatively poor buyers with sufficiently high marginal utility of consumption are willing to pay in cash rather than use electronic payment even if the cash-withdrawing cost exceeds the tax burden associated with the use of electronic payment. However, as the government increases the tax rate on labor, a greater number of poor buyers prefer electronic payment to cash payment, yielding economy of scale in the use of an electronic payment system. This economy of scale decreases per transaction cost of electronic payment, consequently enhancing both intensive-margin and extensive-margin effects of labor taxation.

These results imply that overall transaction cost, including welfare loss because of resource cost for an electronic payment system (which is raised in the form of labor taxation and distortionary consumption taxation) and cash-withdrawing cost, decreases with the tax rate on labor. As a result, the unity tax rate on labor (or zero tax rate on consumption) yields an optimal allocation of the social cost for an electronic payment system.

The paper is organized as follows. Section II describes the model economy. Section III defines a symmetric stationary equilibrium. Section IV formulates the Ramsey problem for the benevolent government and examines an optimal allocation of the social cost for an electronic payment system. Section V summarizes the paper and provides several concluding remarks.

II. Model

The background environment is in line with a standard random matching model of money (*e.g.*, Shi 1995; Trejos and Wright 1995) augmented with distribution of money holdings such as Camera and Corbae (1999), Zhu (2003), Berentsen *et al.* (2005), and Molico (2006). A non-degenerate distribution of monetary wealth is considered to capture economy of scale in the use of an electronic payment system.

Time is discrete. There are $N \geq 3$ number of "islands" with N types of divisible and perishable goods. In each island, there is a [0, 1] continuum of infinitely lived agents. We refer to an agent whose home-island is $n \in \{1, 2, \ldots, N\}$ as a type-n agent. A type-n agent potentially produces only good n, which incurs a disutility cost of $c(q) = q$ for producing q units of good, whereas a type-n agent potentially consumes only good $(n+1)$ (modulo N), which gives a utility of $u(q)$ for consuming q units of a good where $u'' < 0 < u'$, $u(0) = 0$, $u'(0) = \infty$, and $u'(\infty) = 0$. Each agent maximizes expected discounted utility with a discount factor $\beta \in (0, 1)$.

Three exogenous nominal quantities describe the stock of money: upper bound on individual money holdings, size of the smallest unit of money, and average money holdings per type of agent. We normalize the smallest unit to be unity so that the set of possible individual money holdings consists of integer numbers, namely, $\mathbf{M} = \{0, 1, 2, \ldots, M\}$ where $M > 0$ denotes the upper bound required for compactness. We denote the average money holdings per type of agents by $\bar{m} > 0$.

At the beginning of each period, each island is randomly connected to another island, which can be interpreted as a preference shock in the sense that an agent becomes a buyer or a seller with an equal probability $1/N$ or that an agent is neither a buyer nor a seller with the remaining probability $1 - (2/N)$. Now, a seller merely stays on his or her home island, whereas a type-n buyer faces relocation shock so that she should move to a "village" of $j \in \{0, 1, 2, \ldots, M\}$ of $(n+1)$ island, where the village j consists of agents holding j amount of money. For a subsequent pairwise trade in a relocated island, each buyer determines how much to purchase and whether to carry it in cash or deposit it into a checking account. Idle money that is not necessary for immediate subsequent trade can be kept at home without any cost. The government has an intra-temporal record-keeping technology on checking accounts but not on agents' trading histories. Other than the account-related tasks, the government does not engage in any other economic activities, including consumption or production of any goods.

In a newly migrated island, each buyer is matched with a seller for a pairwise trade. Trading histories are private and agents cannot commit to future actions; thus, any possibility of credit trades is ruled out and a medium of exchange is essential (see, for instance, Kocherlakota 1998; Wallace 2001; Corbae *et al.* 2003; Aliprantis *et al.* 2007). That is, either cash or checking-account deposit should be transferred in exchange for goods produced.

In a pairwise trade, a buyer makes a take-it-or-leave-it offer (q, p) to a seller, where q denotes quantity of goods produced by the seller for the buyer and p denotes the amount of money transferred by the buyer to the seller.[1] A buyer can pay p in cash by carrying it to a bilateral meeting at a disutility cost of ηp or withdrawing it from the account at the same disutility cost.[2] Then, even a buyer who is willing to purchase good in cash will deposit the relevant amount of money into a checking account and withdraw it on the spot for a pairwise trade.

Transactions via an electronic payment method require the installment and operation of an electronic payment system, for which the government (*i.e.*, service provider) incurs a fixed cost of Ω. In practice, setting up an electronic payment system entails substantial fixed cost, but trivial marginal cost. In order to focus on the former, the latter is assumed to be zero. From now on, a debit card will be regarded as the representative method of electronic payment because it is one of the primary means of electronic payment for in-store purchases and typically lacks credit function. If a buyer uses a debit card to transfer p amount of money to a seller, the government withdraws it from the buyers's account and transfers it to the seller in cash.

In order to provide debit-card service to the society, the government should raise resources Ω to pay for the fixed cost incurred in installing the debit-card payment system. First, unlike a representative agent model such as Lagos and Wright (2005), our model allows for wealth heterogeneity across agents, including those with no money. Hence, allocating the social cost for the debit-card payment system in a lump-sum fashion is not feasible. Under the assumption that cash-withdrawing cost from a checking account is the same as its carrying cost, taxation on cash traders is not feasible because they would then evade taxation by carrying around cash, which is not in the government information

[1] As shown by Head and Kumar (2005) and Rocheteau and Wright (2005), for instance, this trading protocol can mimic an allocation in a competitive market (or a market with competitive price posting). Suppose that type-n buyers in village j are matched collectively with type-$(n+1)$ sellers in village k and then they trade in a competitive market (or in a market with competitive price posting). An equilibrium allocation would be reminiscent of an allocation with a buyer-take-all bargaining, where a buyer has j amount of money and a seller has k amount of money.

[2] In Section IV, we consider the case in which the proportional cost η is replaced with a fixed cost. We also consider the case in which cash-carrying cost is borne by a seller.

system. However, the government is notified of debit-card transactions because they are cleared through the debit-card payment system operated by the government. Therefore, the government can effectively collect the required resources from debit-card traders.

Specifically, according to the widespread agreement on fixed cost per debit-card transaction (see, for instance, Wang 2010), the government is assumed to raise resource cost Ω by levying ω per debit-card trade where ω satisfies the following government's budget constraint:

$$\Omega = \mathbf{S}\left(\tau\omega + \tau^{c}\omega\right). \tag{1}$$

Here, $\mathbf{S}$ denotes the frequency of debit-card transactions; $\tau \in [0, 1]$ is the share of the cost allocated to a buyer using a debit card; and $\tau^c = (1 - \tau) \in [0, 1]$ is the share of the cost allocated to a seller accepting a debit card. As discussed in the Introduction, the installment cost in the real world is a one-time cost. In our model, however, the government raises Ω in each period. Our interpretation of this is as follows: the government first finances total installment cost Φ and then raises its annuity value $\Omega = (1 - \beta)\Phi$ in each period to smooth out the taxes over time.

For a given (Ω, τ), $\mathbf{S}$ captures the economy of scale in the use of the debit-card payment technology in the sense that a higher $\mathbf{S}$ implies a lower cost per debit-card transaction (ω). We assume that debit-card traders pay the cost to the government by producing their specialized types of goods at the point of a bilateral trade before returning to their home islands. We will show in Section IV (Optimal Cost Allocation) that, with the bargaining rule of a buyer's take-it-or-leave-it offer, the cost share $\tau \in [0, 1]$ allocated to a buyer can be interpreted as taxation on the buyer's labor required to produce output for tax payment. On the other hand, the cost share $\tau^c \in [0, 1]$ allocated to a seller can be interpreted as taxation on the buyer's consumption.

For a type-n agent, per-period utility is given by

$$u(q_{n+1}) - \tilde{q}_n - \tau\omega\mathbf{I} - \eta c(1 - \mathbf{I}) \tag{2}$$

where q_{n+1} is the consumption of good $n+1$, $\tilde{q}_n$ is the production of good n (including the share of cost $\tau^c\omega$ borne as a seller by accepting debit cards); $\mathbf{I}$ is an indicator function that is equal to 1 if a type-n agent as a buyer meets a type-$(n+1)$ agent and trades via a debit card; and c is the amount of cash withdrawn by a type-n agent for a pairwise trade.

After a pairwise trade, a buyer returns to her home island and the information on each checking account is completely wiped out.[3] An agent goes on to the next period with end-of-period money holdings.

III. Equilibrium

We study a symmetric (across "islands" or specialization types) and stationary equilibrium. For a given (Ω, η) and government policy on τ, a symmetric steady state consists of functions (v, π) and ω that satisfy the conditions described in the following. The functions v: $\mathbf{M} \rightarrow \mathbf{R}$ and π: $\mathbf{M} \rightarrow [0, 1]$ pertain to the beginning of a period and prior to the realization of preference shock such that $v(m)$ is the expected discounted value of having monetary wealth m and $\pi(m)$ is the fraction of each island with monetary wealth m.

Consider a generic pairwise meeting between a buyer with $b \in \mathbf{M}$ and a seller with $s \in \mathbf{M}$. We let

$$\Gamma(b, s) = \left\{ p : p \in \{0, 1, ..., \min\{b, M - s\}\} \right\}. \tag{3}$$

That is, $\Gamma(b, s)$ is the set of feasible wealth transfers from the buyer to the seller. After the realization of relocation shock, the buyer determines how much to offer (p) to the seller and how to pay it, cash $(\mathbf{I}=0)$ or debit card $(\mathbf{I}=1)$. Noting that the seller accepts all offers that leave her no worse off (tie-breaking rule), the buyer's problem can be expressed as

$$\max_{\mathbf{I} \in \{0,1\}} \left\{ \mathbf{I} \left[\max_{p \in \Gamma(b,s)} v_d(b) \right] + (1 - \mathbf{I}) \left[\max_{p \in \Gamma(b,s)} v_c(b) \right] \right\} \tag{4}$$

where

$$v_d(b) = u \left\{ \beta v \left[s + p(b, s, v) \right] - \beta v(s) - \tau^c \omega \right\} + \beta v \left[b - p(b, s, v) \right] - \tau\omega \tag{5}$$

[3] Note that unsecured credit transactions are available and money is inessential if the government is assumed to have an inter-temporal record-keeping technology rather than an intra-temporal one.

$$v_c(b) = u\left\{\beta v\left[s + p(b, s, v)\right] - \beta v(s)\right\} + \beta v\left[b - p(b, s, v)\right] - \eta p(b, s, v). \quad (6)$$

Let $p_d(b, s, v)$ and $p_c(b, s, v)$ be the solutions to the first bracket and second bracket of the right-hand side in (4), respectively; that is,

$$p_d(b, s, v) = \arg \max_{p \in \Gamma(b,s)} \left\{u\left[\beta v(s + p) - \beta v(s) - \tau^c \omega\right] + \beta v(b - p) - \tau\omega\right\} \quad (7)$$

$$p_c(b, s, v) = \arg \max_{p \in \Gamma(b,s)} \left\{u\left[\beta v(s + p) - \beta v(s)\right] + \beta v(b - p) - \eta p\right\}. \quad (8)$$

Now, let $p^*(\cdot)$ be the associated $p_i(\cdot)$ for $i \in \{c, d\}$ such that $p^*(b, s, v) = p_d(b, s, v)$ if the buyer chooses to pay using a debit card ($I = 1$) and $p^*(b, s, v) = p_c(b, s, v)$ if the buyer chooses to pay in cash ($I = 0$).[4] $p(b, s, v)$ is discrete and thus, $p^*(b, s, v)$ can be multi-valued, in which case we allow for all possible randomizations over them. Let $\Delta(b, s, v)$ be the set of measures that represents those randomizations. Then $\Delta(b, s, v)$ can be described as

$$\Delta(b, s, v) = \{\delta(\cdot; b, s, v) : \delta(m; b, s, v) = 0 \ \text{ if } m \notin \{b - p^*(b, s, v)\} \quad (9)$$

where $\delta(m; b, s, v)$ is the probability that in a pairwise trade, the buyer with $b \in \mathbf{M}$ offers $b - m$ to the seller with $s \in \mathbf{M}$, ending up with m.

Now we can describe the evolution of wealth distribution induced by pairwise trades as follows:

$$\Pi(v) = \left\{\pi : \pi(m) = \frac{1}{N} \sum_{(b,s)} \pi(b)\pi(s)[\delta(m) + \delta(b - m + s)] \right.$$
$$\left. + \frac{N-2}{N} \pi(m) \text{ for } \delta \in \Delta(b, s, v)\right\}. \quad (10)$$

The first probability measure on the right-hand side of (10) corresponds to pairwise trades, whereas the second corresponds to all other cases.

[4] Without the loss of generality, we can disregard the case in which a fraction of p is paid in cash and the remainder using a debit card because once a debit card is used, cost per debit-transaction (ω) is imposed regardless of the amount of debit-card transaction.

Noting that in (9), δ is defined over the post-trade money holdings of the buyer, the buyer's post-trade money holdings $(b-m+s)$ corresponds to the seller's post-trade wealth $m(=b-(b-m+s)+s)$. Note also that the dependence of Π on v comes from the dependence of δ on v in (9).

Finally, let $g(b, s, v)$ be the maximized value of (4). Note that the payoff of a seller with $m \in \mathbf{M}$ is simply $\beta v(m)$, and thus, the expected value of holding m at the beginning of a period, $v(m)$, can be written as

$$v(m) = \frac{1}{N}\sum_{s}\pi(s)g(m, s, v) + \frac{N-1}{N}\beta v(m). \tag{11}$$

Definition 1: *For given* $(\Omega, \eta\ \tau)$, *a symmetric stationary equilibrium is a set of functions* (v, π) *and* ω *such that (i) the value function* v *satisfies (11); (ii) the probability measure* π *of wealth distribution satisfies* $\pi \in \Pi(v)$ *where* $\Pi(v)$ *is given by (10); and (iii)* ω *satisfies the government's budget constraint,* $\Omega = \omega\sum_{(b,s)}\pi(b)\pi(s)\mathbf{I}(b, s, v)$.

The existence of a symmetric stationary equilibrium for some parameters is a straightforward extension of the existence results in Zhu (2003) and Lee *et al.* (2005). That is, if $(\bar{m}, M/\bar{m})$ are sufficiently large respectively and (Ω, η) are not too large respectively, then a monetary symmetric stationary equilibrium (v, π, ω) exists with v strictly increasing and strictly concave.

IV. Optimal Cost Allocation

We now examine an optimal allocation of the social cost for an electronic payment system using a standard Ramsey taxation approach.

A. The Ramsey Problem

In order to formulate a Ramsey problem for the benevolent government as a provider of debit-card payment services, we first define welfare as the lifetime expected discounted utility of a representative agent before the assignment of wealth according to a stationary distribution. Let $\mathbf{W}_\tau$ denote the welfare of a stationary equilibrium $(v_\tau, \pi_\tau, \omega_\tau)$ for a given policy τ. Then $\mathbf{W}_\tau$ can be expressed as follows:

$$\mathbf{W}_{\tau} = \frac{\pi_{\tau} \mathrm{U} \pi'_{\tau}}{(1-\beta)N}. \quad (12)$$

Here, the element in row $b \in \mathbf{M}$ and column $s \in \mathbf{M}$ of the matrix $\mathbf{U}$ is

$$u[q(b, s, v_{\tau})] - \tilde{q}(b, s, v_{\tau}) - \tau\omega \mathbf{I}(b, s, v_{\tau}) - \eta c(b, s, v_{\tau})[1 - \mathbf{I}(b, s, v_{\tau})]$$

with $c(\cdot)$ denoting the amount of cash withdrawn for a pairwise trade $\tilde{q}(\cdot) = q(\cdot) + \tau^{c}\omega \mathrm{I}(\cdot)$ and where the second term, $\tau^{c}\omega \mathrm{I}(\cdot)$, captures the disutility from the production of output for taxation borne as a seller by accepting debit cards.

Under the buyer-take-all trading protocol, the benefit principle is implementable in the sense that the social cost for the debit-card payment system is borne by its beneficiaries (*i.e.*, buyers) regardless of τ. That is, from (4), in a pairwise trade between a buyer with $b \in \mathbf{M}$ and a seller with $s \in \mathbf{M}$, the buyer's net payoff for $\mathbf{I}=1$ is

$$u[q(b, s, v_{\tau}) - \tau^{c}\omega] - \tau\omega + \beta\{v_{\tau}[b - p_{d}(b, s, v_{\tau})] - v_{\tau}(b)\} \quad (13)$$

where $q(b, s, v_{\tau}) = \beta v_{\tau}(s + p_{d}) - \beta v_{\tau}(s)$ and $\tau + \tau^{c} = 1$. Therefore, the choice of τ can be interpreted as allocating the social cost to the beneficiaries of debit-card transactions in the form of labor taxation ($\tau = 1$, $\tau^{c} = 0$) or consumption taxation ($\tau = 0$, $\tau^{c} = 1$) or a certain combination of the two taxations [τ, $\tau^{c} \in (0, 1)$]. In the above equation, labor taxation ($\tau = 1$, $\tau^{c} = 0$) has a lump-sum feature in the sense that it does not affect quantity consumed in exchange for money transferred via a debit card. On the other hand, the consumption taxation ($\tau = 0$, $\tau^{c} = 1$) is distortionary in the sense that it affects quantity consumed with the debit-card transactions.

Now the Ramsey problem for the government is to choose the tax rate on labor (τ) and the implied tax rate on consumption ($\tau^{c} = 1 - \tau$) to maximize welfare, taking into account its effect on the equilibrium reactions of buyers and sellers in pairwise trades.

Definition 2: *The Ramsey problem for the benevolent government is to choose a symmetric stationary equilibrium* (v_{τ}, π_{τ}, ω_{τ}) *in Definition 1 that maximizes (12) or equivalently choose* $\tau^{*} = \arg\max_{\tau \in [0, 1]} \mathbf{W}_{\tau}$.

In order to compare the magnitude of welfare loss across different policies, we calculate the welfare cost of τ-policy relative to that of an infeasible lump-sum taxation.[5] More specifically, we first find Δ that solves

$$\bar{\mathbf{W}} = \frac{\pi_\tau \mathrm{U}_\Delta \pi'_\tau}{(1-\beta)N} \tag{14}$$

where $\bar{\mathbf{W}}$ is the welfare with lump-sum taxation. The element in row $b \in \mathbf{M}$ and column $s \in \mathbf{M}$ of U_Δ is

$$u[q(b, s, \upsilon_\tau) + \Delta] - \tilde{q}(b, s, \upsilon_\tau) - \tau\omega\mathbf{I}(b, s, \upsilon_\tau) - \eta c(b, s, \upsilon_\tau)[1 - \mathbf{I}(b, s, \upsilon_\tau)]\,.$$

That is, Δ is an additive consumption compensation that makes the welfare with τ-policy equal to that with lump-sum taxation ($\bar{\mathbf{W}}$).[6] The welfare cost of τ-policy is then calculated as the ratio of Δ to the average consumption in the stationary equilibrium with τ-policy.

Although labor taxation (τ) has a lump-sum feature as in (13), whether a higher τ improves or deteriorates welfare is not obvious at all. The marginal gain from a higher τ is $(\partial u/\partial\tau) = u'(q)\omega$, whereas the marginal labor cost of increasing τ is just ω. Hence, for a given ω a higher τ is beneficial to relatively poor buyers whose consumption is sufficiently small so that $u'(q) > 1$, whereas it is detrimental to relatively rich buyers whose consumption is sufficiently large so that $u'(q) < 1$. This implies that relatively poor buyers prefer labor taxation, whereas relatively rich buyers prefer consumption taxation. Notice that, for a given ω, a higher τ increases the intensive margin (output per unit of money), which then increases the fraction of buyers with $u'(q) < 1$.

Labor taxation (τ) also has an extensive-margin effect on the choice between electronic and cash payment. For a simple illustration of this point, consider a pairwise trade between a buyer and a seller who have money holdings of b and s, respectively. Suppose money is divisible and

[5] If the social cost for debit-card payment system is raised by imposing a lump-sum tax on each agent, all trades are made using debit cards, and trading behaviors are not affected. However, as discussed in Section II, lump-sum tax cannot be forced effectively in our model.

[6] Consumption compensation is calculated as an addition to the consumption with τ-policy rather than as a multiple of consumption with τ-policy, because consumption is zero in some pairwise meetings.

the value function v is differentiable. Now, for given (η, ω, τ), let $\bar{p}(b, s)$ denote the amount of money transfer at which a buyer is indifferent between cash and debit-card payment; that is,[7]

$$u\left\{\beta[v(s+\bar{p}(b, s))-v(s)]\right\}-u\left\{\beta[v(s+\bar{p}(b, s))-v(s)]-(1-\tau)\omega\right\} \\ +\tau\omega=\eta\,\bar{p}(b, s). \tag{15}$$

It can be shown that $\bar{p}(b, s)$ decreases with τ and debit-card payment is preferred for $p(b, s)>\bar{p}(b, s)$. This is particularly true of relatively poor buyers with $u'(q)>1$. Hence, as τ increases, a greater number of poor buyers are willing to use debit cards, and the fraction of debit-card traders with $u'(q)>1$ would consequently increase.

This extensive-margin effect will lower per transaction cost (ω) of the debit-card payment system via an economy-of-scale channel, which in turn will enhance both intensive and extensive margin effects of a higher τ. For instance, a lower ω not only increases output produced per unit of money (*i.e.*, intensive margin) but also decreases $\bar{p}$ in (15), implying that a larger fraction of buyers use debit-card payment (*i.e.*, extensive margin).

The above discussion suggests that the effect of τ on welfare depends crucially on a nondegenerate distribution of wealth across agents and economy of scale in the use of debit-card payment system. The endogeneity of a nondegenerate wealth distribution rules out closed-form solutions and hence, we tackle our question based on numerical solutions in the following subsections.

B. Parameterization

We parameterize the basic environment to solve the model numerically as follows. We first assume $N=3$, the smallest number of types of agents consistent with no double-coincidence meeting, and $\beta=0.99$. We set $(\bar{m}, M)=(40, 3\bar{m})$ to ensure that the indivisibility of money and the upper bound on money holdings are not too severe.[8]

[7] Note that the left-hand side of (15) is decreasing in p, whereas the right-hand side is increasing in p. The left-hand side is also greater than the right-hand side when p is sufficiently close to 0. Hence, $\bar{p}(b, s)$ in (15) is well defined.

[8] In this type of model, almost all monetary offers are either 0 or 1 if the indivisibility of money is too severe. However, it is not the case in our examples. In addition, $M=3\bar{m}$ is large enough so that almost no one is at the upper bound in a stationary equilibrium and hence, the result would be hardly affected even

We let $u(q)=\theta\ln(1+q)$ where θ is chosen, together with (Ω, η), to make the model fit the U.S. data. Based on the 2011 Survey of Consumer Payment Choice, Shy (2012) reports that $\mathbf{S}=0.511$ where $\mathbf{S}$ is the fraction of debit-card transactions out of cash and debit-card transactions. In addition, based on the 2010 Diary of Consumer Payment, Stavins (2012) reports that the average value of debit-card transactions relative to that of cash, denoted by $\mathbf{D}$, equals to 1.65. In our model, $(\theta, \Omega, \eta)=(2.1, 2.0\times10^{-3}, 8.42\times10^{-4})$ with $\tau=0$ generates $\mathbf{S}=0.512$ and $\mathbf{D}=1.50$. Here, we consider $\tau=0$ because in the U.S. almost all costs of debit-card transactions are imposed on the sellers on the surface. Note that $\Omega=2.0\times10^{-3}$ corresponds to 0.18% of $q^*=\arg\max\,[u(q)-q]=1.1$, which is close to the estimate of Aiyagari *et al.* (1998).

C. Labor Taxation versus Consumption Taxation

Figure 1 shows welfare level and welfare cost as a function of the tax rate on labor (τ). The welfare increases with τ and the solution to the Ramsey problem turns out to be $\tau^*=1$. That is, the unity tax rate on labor (or zero tax rate on consumption) attains the highest welfare. The welfare cost with zero tax rate on consumption ($\tau=1$) is 0.03%, but increases up to 0.12% with zero tax rate on labor ($\tau=0$).

The underlying mechanism that renders the unity tax rate on labor (or zero tax rate on consumption) optimal can be explained in terms of (i) its intensive-margin effect on the quantity consumed in exchange for money transferred via a debit card for a given cost per debit transaction and (ii) its extensive-margin effect on the choice of means of payment between debit card and cash, which affects the cost per debit transaction. The welfare implications of these two effects hinge on a nondegenerate distribution and economy of scale in the use of debit-card payment system.

First, with regard to the intensive-margin effect, a higher tax rate on labor implies relatively low tax rate on consumption, thereby mitigating consumption distortion. Hence, the higher the tax rate on labor (consumption), the more favorable to the relatively poor (rich) whose consumption is sufficiently small (large). Figure 2 shows that the gap between the average quantity of good consumed by buyers and that produced by sellers shrinks as the tax rate on labor increases. As a result, welfare loss due to distortionary consumption taxation decreases and

if a larger M were assumed.

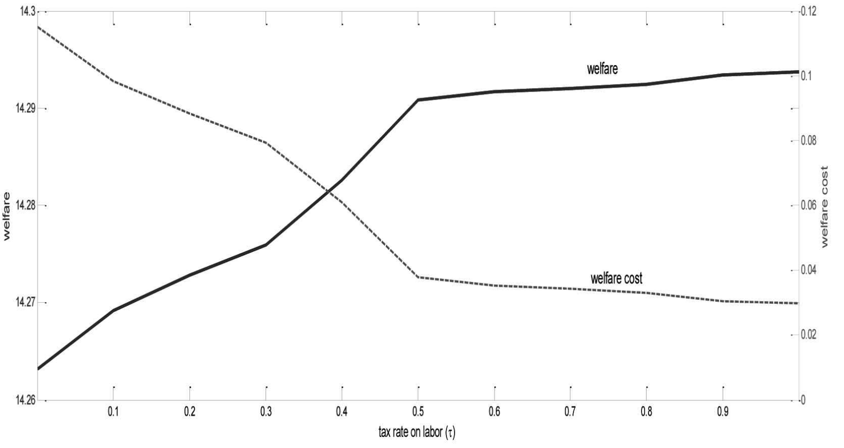

FIGURE 1
WELFARE LEVEL AND WELFARE COST

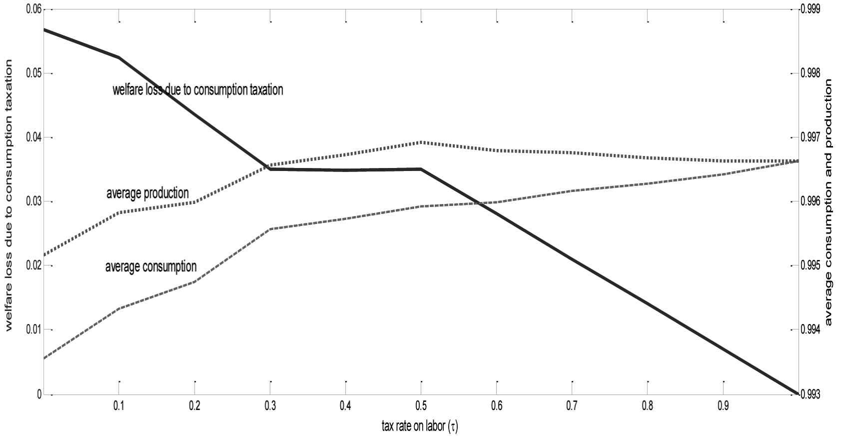

FIGURE 2
WELFARE LOSS DUE TO CONSUMPTION TAXATION

eventually converges to zero as the tax rate on labor approaches 1.

Second, a higher tax rate on labor also has an extensive-margin effect on the choice of payment method. Figure 3 shows that the frequency of debit-card transactions (**S**) increases with τ, which eventually lowers cost per debit-card transaction ($\omega = \Omega/\mathbf{S}$). The two extreme policies, $\tau = 1$ and $\tau = 0$ for a given (ω, η), are useful in understanding the frequency

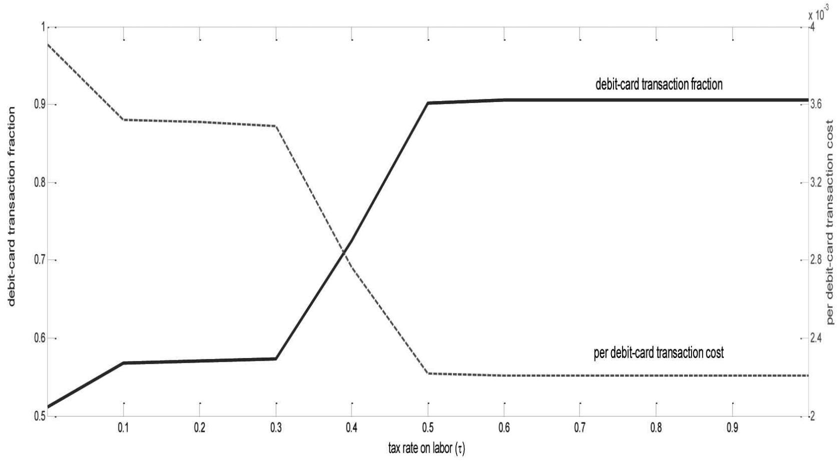

FIGURE 3
FREQUENCY OF DEBIT-CARD TRANSACTIONS AND COST PER TRANSACTION

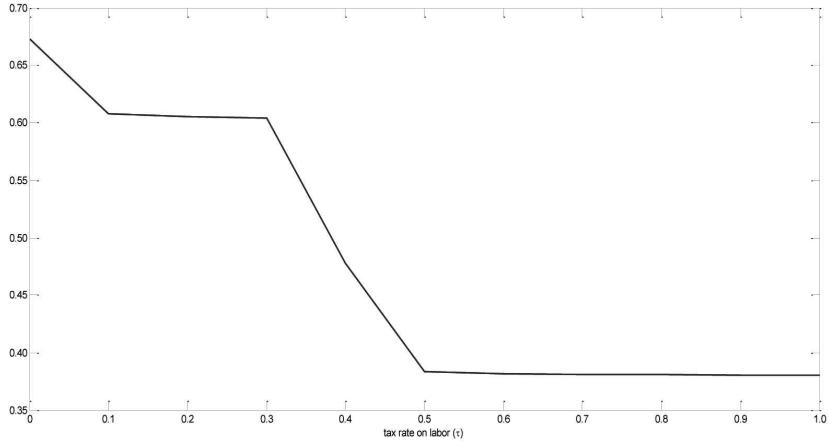

FIGURE 4
FRACTION OF BUYERS WITH $u'(q)<1$

of debit-card transactions (S) increases with τ. In the case of $\tau=1$, all transactions are completed using debit cards as long as $p>(\omega/\eta)$. However, when $\tau=0$, transactions can be made in cash even if $p>(\omega/\eta)$. That is, p exceeding (ω/η) will be paid in cash rather than using a debit card as long as $u\{\beta v[s+p(\cdot)]-\beta v(s)\}-u\{\beta v[s+p(\cdot)]-\beta v(s)-\omega\}>$

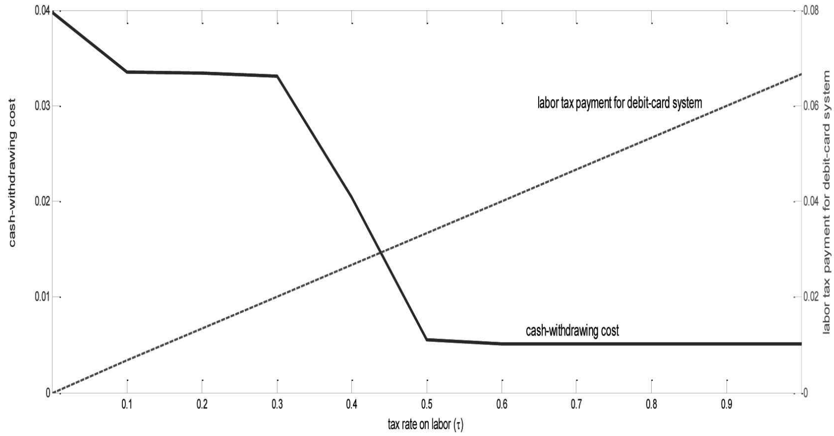

FIGURE 5
CASH-WITHDRAWING COST AND LABOR TAX PAYMENT FOR A DEBIT-CARD SYSTEM

$\eta p(\cdot)$. The latter happens to a relatively poor buyer in a pairwise trade when $\beta v[s+p(\cdot)]-\beta v(s)\equiv q_c$ is sufficiently small. A higher frequency of debit-card transactions with a higher tax rate on labor enhances economy of scale as it subsequently lowers cost per debit-card transaction, as shown in Figure 3. Also, Figure 4 shows that the extensive-margin effect reduces the fraction of buyers with $[u'(q)<1]$ who prefer consumption taxation.

Third, Figure 5 shows that the total cost of withdrawing cash decreases with the tax rate on labor. This is because the frequency of cash transactions decreases as τ increases. More transactions are subject to labor taxation and hence, the cost for the debit-card payment system allocated in the form of labor tax also increases with the tax rate on labor.

In sum, Figure 6 shows that overall transaction cost, including welfare loss due to social cost for the debit-card payment system (which is raised in the form of labor taxation and distortionary consumption taxation) and cash-withdrawing cost, decreases with the tax rate on labor. This immediately implies the results shown in Figure 1.

Finally, we check the robustness of our main results in different settings. We first consider the fixed cash-carrying cost for sellers. That is, a seller accepting cash incurs a fixed handling cost κ.[9] In computing a stationary equilibrium for this case, we set κ to make the model fit

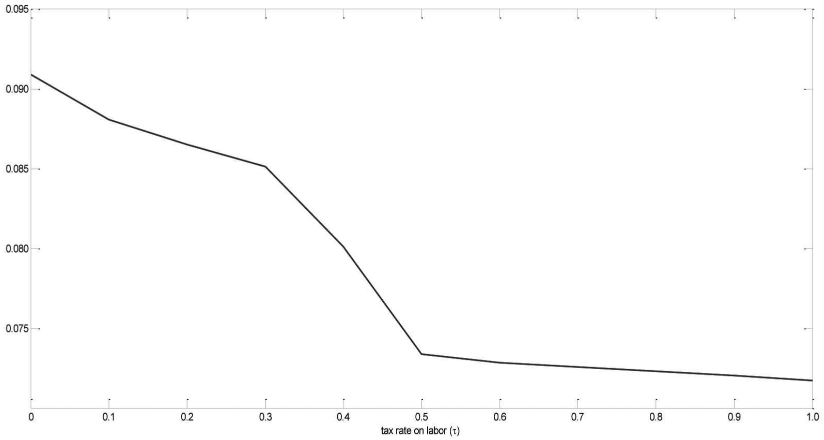

FIGURE 6
OVERALL TRANSACTION COST

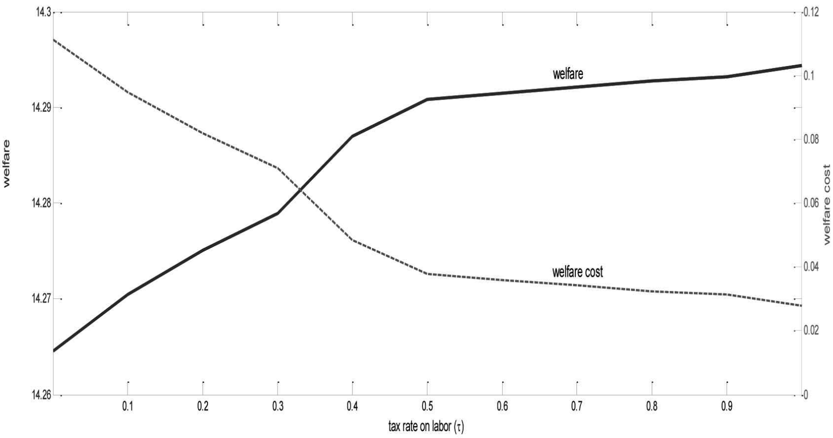

FIGURE 7
FIXED CASH-CARRYING COST FOR SELLERS

the U.S. data on the fraction of debit-card transactions (S=0.511) and the ratio of average value of debit-card transactions to that of cash (D= 1.65). The model parameterized with $(\theta, \Omega, \eta, \kappa)=(2.1, 2\times10^{-3}, 8\times10^{-4},$

[9] Recent empirical studies such as Garcia-Swartz *et al.* (2006) and Schmiedel *et al.* (2012) suggest that both buyers and sellers incur cash-carrying cost.

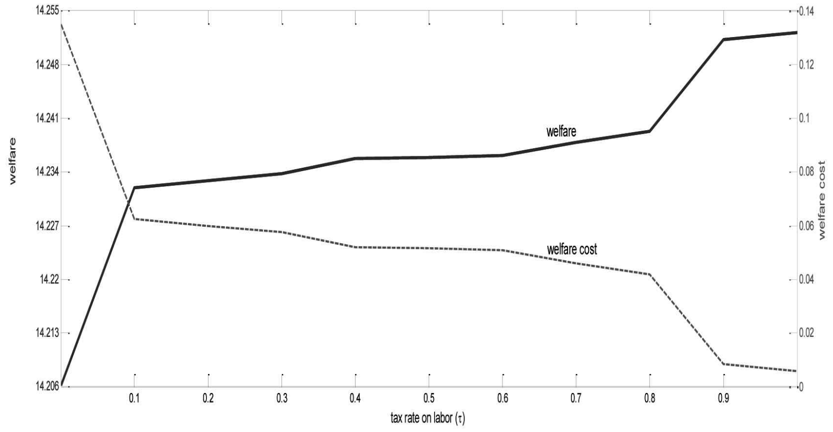

FIGURE 8
FIXED CASH-WITHDRAWING COST AND PROPORTIONAL CASH-CARRYING COST

1.15×10^{-4}) implies **S**=0.512 and **D**=1.50 As reported in Figure 7, this variation does not change the optimality of the unity tax rate on labor.

As another robustness check, we consider the case in which the cash-withdrawing cost for a buyer is fixed, whereas the cash-carrying cost for a seller is proportional. That is, withdrawing cash incurs a fixed cost $\bar{\eta}$ for a buyer, whereas cash-handling cost for a seller increases with the amount of cash transaction at a rate of $\tilde{\kappa}$. In computing a stationary equilibrium for this case, we again choose $(\bar{\eta}, \tilde{\kappa})$ to make the model fit the data. The model parameterized with $(\theta, \Omega, \bar{\eta}, \tilde{\kappa})=(2.1, 3.5\times10^{-3}, 1.01\times10^{-3}, 1\times10^{-3})$ implies **S**=0.568 and **D**=1.56. Figure 8 shows that optimality of the unity tax rate on labor is also immune to this variation.

V. Concluding Remarks

In this paper, we explored an optimal allocation of the social cost for an electronic payment system by incorporating a standard Ramsey taxation approach into an off-the-shelf matching model of money. Our results suggest that economy of scale in the use of an electronic payment technology is enhanced with the tax rate on labor. This then decreases not only per transaction cost of electronic payment and cash-withdrawing cost, but also welfare loss because of the distortionary consumption taxation. As a result, the unit tax rate on labor (or zero tax rate on consumption) yields an optimal allocation of the cost for an electronic payment system.

In order to capture an economy-of-scale channel thoroughly, we here adopt a Trejos-Wright model. Alternatively, we can also consider a rather tractable model such as a version of Lagos and Wright (2005). For the model, tractable distribution of wealth can be generated in many ways, allowing us to obtain a closed-form solution. We believe, however, that even in this case, our main mechanism would still work and the main results would remain intact.

Finally, we focused on an allocation scheme of the social cost for an electronic payment system without considering its implications from the viewpoint of an industrial organization. For instance, we do not deal with the issue on how to impose fees on merchants and consumers by a profit-maximizing card issuer. We leave the generalization of our model to future research, in which private fees together with the resource cost can be explored systematically.

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