New Empirical Evidence for the Fisher Relation: Integration and Short-run Instability

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The Fisher relation is a key theoretical relation that underlies many important results in economics and finance. Alhough the Fisher relation is apparently simple in theory, empirical analyses of the relation have mixed and weak results. We consider the possibility that weakness of the evidence is due to short-run instability in the relation, which is sufficiently strong to dominate the whole sample. We analyze this possibility based on the following two approaches. First, we apply partial-sample instability tests of Andrews and Kim (2006) to detect such short-run instability. Our result shows clear evidence for the existence of such short-run instability. Second, we examine how much the partial-sample instability affects the long-memory property of the real interest rate based on the concept of fractional integration. Our result indicates that the short-run instability causes a substantial increase in the coefficient of fractional integration, which implies an increase in the tendency of nonstationarity.

Keywords: Fisher relation, Short-run instability, Fractional integration

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I. Introduction

The Fisher relation is a key theoretical relation that underlies many important results in economics and finance. This relation describes that the nominal interest rate has a stable one-for-one relation with the expected rate of inflation. Thus, the Fisher relation implies a constant or stable level of the "real interest rate" that is equal to the nominal interest rate minus expected inflation. Typical economic theory assumes that the real interest rate is a constant or a stationary variable fluctuating around a constant mean, as implied in the Fisher hypothesis. Although the Fisher relation looks a simple relation, the empirical analysis is somewhat complicated with mixed results. In this paper, we examine the Fisher relation and related issues for data from the U.S. and Korea based on some recently developed econometric methods.

Since Fama (1975) pioneered the empirical work on the Fisher relation, many researchers have investigated data for the Fisher relation. The hypothesis that the real interest is constant was studied by Nelson and Schwert (1977), Garbade and Wachtel (1978), Mishkin (1981, 1984), and Fama and Gibbons (1982). Correlation between the inflation rate and a nominal interest rate (Fisher effect) was studied by Nelson and Schwert (1977), as well as Fama and Gibbons (1982), Summers (1982), Huizinga and Mishkin (1986), and Mishkin (1990). An alternative approach to the empirical Fisher relation has been used by Rose (1988), Atkins (1989), Mishkin (1992), and Wallace and Warner (1993) based on the concepts of unit roots and cointegration. Nominal interest rates and inflation usually have nonstationary properties (Crowder and Hoffman 1996). In such a situation, data support the Fisher relation if the real interest rate is stationary (Mishkin 1992). Often, however, stationarity of the real interest rate is not well confirmed by U.S. data (Rose 1988; Walsh 1987).

In this paper, we examine why the existing evidence in favor of the Fisher relation is weak and mixed, especially in works based on the concepts of unit roots and cointegration. Our analysis is based on the conjecture that weakness of the evidence is due to short-run instability in the relation. We analyze this conjecture based on the following two approaches. First, we apply the partial-sample instability tests of Andrews and Kim (2006) to detect such short-run instability. Our result shows clear evidence for the existence of such short-run instability for data from the U.S. and Korea in the postwar era. Second, we examine how

much the partial-sample instability affects the long-memory property of the real interest rate based on the concept of fractional integration. A higher fractional integration (longer memory) implies a higher tendency of nonstationarity. Our result indicates that short-run instability causes a substantial increase in the coefficient of fractional integration, which implies an increase in the tendency of nonstationarity.

The paper is organized as follows. Section II introduces the Fisher relation and related issues to be studied in this paper. Section III explains data used in the paper. In Section IV we analyze partial-sample instability of the Fisher relation; in Section V, the fractional integration property of the real interest rate is examined. Section VI concludes the paper.

II. The Fisher Relation and Related Issues

The Fisher relation explains how the interest rate is determined. It describes that the nominal interest rate has a stable one-for-one relation with the expected rate of inflation. In other words, the Fisher relation describes a stable level of the "real interest rate" that is equal to the nominal interest rate minus expected inflation. In terms of *ex-ante* variables, the relation is written as

$$r_t^* = i_t - \pi_{t+1}^e \tag{1}$$

where π_{t+1}^e is the expected inflation from period t to period $t+1$; r_t^* and i_t are the *ex-ante* real interest rate and the nominal interest rate at time t, respectively. In terms of *ex-post* variables, the Fisher relation becomes

$$r_t = i_t - \pi_{t+1} \tag{2}$$

where π_{t+1} and r_t are, respectively, *ex-post* inflation and *ex-post* real interest rate.

Notice that for v_t such that $v_t = \pi_t^e - \pi_t$ we have $r_t = r_t^* - v_t$. Thus, if the error of the inflation expectation v_t is a stationary variable, which is the case under rational expectations, then the *ex ante* real interest rate r_t^* and the *ex post* real interest rate r_t have the same degree of integration. In this case one can analyze the Fisher relation based on the *ex post* interest rate as well as the *ex ante* rate.

As a stationary variable has some stability properties, we sometimes use the term "stability" for stationarity in the following discussions.

Constancy or stationarity/stability of the real interest rate implies the existence of a stable Fisher relation. Thus, typical economic theory assumes that the real interest rate is a constant, or a stationary variable fluctuating around a constant mean. Examples are models of dynamic optimization and intertemporal decision making that are used widely in economics and finance.

However, the empirical analysis of the Fisher relation has been somewhat complicated with mixed results, although the theoretical Fisher relation looks a simple one. One reason for the complication in empirical work is that (nominal) interest rates and/or inflation often show nonstationary properties (Crowder and Hoffman 1996, for example). In such a situation, data support the Fisher relation if the real interest rate is a stationary variable (Mishkin 1992). However, stationarity of the real interest rate is often not well confirmed by real data (Rose 1988; Walsh 1987).

Therefore, we investigate stationarity/stability of the real interest rate based on certain relevant alternative concepts. This subject is very important because stability of the real interest rate is an essential element of many theories in economics and finance. Section IV studies this subject based on the idea that the weakness of evidence in favor of the Fisher relation may be due to short-run nonstationary deviations from the Fisher relation, whereas the relation prevails in the other data periods. To analyze this possibility, we apply the partial-sample cointegration breakdown test in Andrews and Kim (2006). In addition, we investigate the "degree" or fraction of integration for the real interest rate in Section V. The analysis of fractional integration would reveal more detailed information on the properties of a stochastic process regarding stationarity/nonstationarity than does the analysis of the $I(1)$/$I(0)$ property. To analyze fractional integration of the real interest rate we apply the inference method in Phillips (2005).

III. Data and Some Basic Properties

As in many existing works, we use the three-month treasury bill rate or equivalence for the nominal interest rate and the consumer price index (CPI) for the price level to compute the inflation rate. We obtain the U.S. data of the T-bill rate and the CPI from the Federal Reserve Board and the Bureau of Labor Statistics, respectively. Korean data are from the International Financial Statistics (IFS). All data are seasonally

TABLE 1
ADF t-TEST

	Nominal rate	Inflation rate	Real rate
U.S.	-1.84	-2.47	-2.48
Korea	-1.54	-1.80	-2.87

Note: The 5% critical value is -2.89 for $T = 100$.

adjusted. The data period is 1953:Q1-2004:Q1 for the U.S. data and 1976:Q3-2004:Q1 for Korean data.

Given that the stationary and nonstationary ($I(0)/I(1)$) property of a variable is relevant in our analysis, we examine whether the variables contain a unit root. We test the null of a unit root for all the three variables, a nominal interest rate, an inflation rate, and the corresponding real interest rate. The results of the augmented Dickey-Fuller (ADF) t-test are in Table 1.

Table 1 shows that the unit root null is not rejected at 5% for the nominal interest rate and the inflation rate. In addition, the unit root null is not rejected for the real interest rate for both countries. Here, the real interest rate is the *ex post* rate. If the error of the inflation expectation is a stationary variable, which is the case under rational expectations, then the *ex ante* real interest rate and the *ex post* real interest rate have the same degree of integration. In this case, one can analyze the Fisher relation based on the *ex post* interest rate as well as the *ex ante* rate. Thus, the result in Table 1 implies that the Fisher relation explained above is not confirmed for data from the two countries.

IV. Analysis of Partial-sample Instability

In Section III we have found that the Fisher relation is not confirmed for data from the U.S. and Korea: The unit root null is not rejected for the real interest rate at the 5% level. We conjecture that this result is due to short-run instability in the Fisher relation. That is, there may be relatively short period(s) in the sample that contains some important factors causing nonstationarity or instability. In such a case, the short-run instability may be sufficiently strong to dominate the entire sample. We analyze this possibility in this and the next sections.

Consider a version of the Fisher relation in ex-post terms:

$$i_t = c + \pi_t + u_t \tag{3}$$

If $\{u_t\}$ is a stationary process while the nominal rate i_t and the inflation rate π_t are unit root processes, the two variables i_t and π_t are cointegrated with cointegrateion vector (1, -1). In this case, $c+u_t$ is the real interest rate that shows stable fluctuations around a constant c. In addition, in this case, the nominal interest rate and the inflation rate have a stationary one-for-one relation, as the Fisher relation implies. Let $r_t = i_t - \pi_t$, or $r_t = c + u_t$. Suppose that for some reason the real interest rate r_t has nonstationary properties in the m time periods $t = t_0, \ldots, t_0 + m - 1$, whereas it is a stationary process in the other part of the sample period. In this case, the two variables i_t and π_t have a *segmented cointegration* relation in the concept of Kim (2003). Kim (2003) shows theoretically and by simulation that, in the case of segmented cointegration, often the cointegration is likely not well confirmed although a cointegration relation prevails in the majority of the sample period.

We can formalize this situation by the following hypotheses

$$\begin{aligned} &H_0: \{c_t = c_0 \text{ and } \{u_t: t = 1, \ldots, T\} \text{ is stationary and ergodic} \\ &H_1: \begin{cases} c_t \neq c_0 \text{ for some } \{t = t_0, \ldots, t_0 + m - 1\} \text{ and/or} \\ \text{the distribution of } \{u_t: t = t_0, \ldots, t_0 + m - 1\} \text{ differs from} \\ \text{the distribution of } \{u_t: t = 1, \ldots, t_0 - 1, t_0 + m \ldots, T\} \end{cases} \end{aligned} \tag{4}$$

The null hypothesis H_0 describes that the real rate has stationary fluctuations around the mean c_0 in the whole sample period. Under H_1, however, such a stable relation breaks down in m time periods when the real interest rate has unstable/nonstationary fluctuations. In other words, a cointegration relation between i_t and π_t breaks down in the m time periods. Note that the cointegration breakdown in this case may occur when the mean c changes or when the error process has different properties in the m periods.

Andrews and Kim (2006) have proposed inference procedures for analyzing such a situation. We use the P-test in Andrews and Kim (2006) for our analysis. The essential part of the method is as follows: Let P be the sum of square of the residuals in the period of possible temporary instability $\{t = t_0, \ldots, t_0 + m - 1\}$

$$P = P_{t_0}(\hat{c}) = \sum_{t=t_0}^{t_0+m-1} \hat{u}_t^2$$

TABLE 2
THE BREAKDOWN PERIODS IDENTIFIED BY P-TEST

U.S.	1980:Q2 - 1986:Q1
Korea	1978:Q1 - 1981:Q3

where $\hat{u}_t = i_t - \pi_t - \hat{c}$ for the least square estimator of $\hat{c}$. Also, let P_j be the similar one for the other (stationary) periods j

$$P_j = \sum_{t=j}^{j+m-1} \hat{u}_t(\hat{c}_{(j)})^2$$

where $\hat{u}_t(\hat{c}_{(j)}) = i_t - \pi_t - \hat{c}_{(j)}$ for an estimator $\hat{c}_{(j)}$. For an estimator $\hat{c}_{(j)}$ in P_j Andrews and Kim (2006) recommend using the "leave-$m/2$-out" estimator, $\hat{c}_{2(j)}$,

$\hat{c}_{2(j)}$ = estimator of c using observations indexed by $t = 1, \ldots, T \setminus \{t_0, \ldots, t_0 + m - 1\}$
with $t \neq j, \ldots, j + [m/2] - 1$

where $[m/2]$ denotes the smallest integer that is greater than or equal to $m/2$. Then, the decision in the procedure is based on a p-value computed by the following

$$pv_p = (T - m + 1)^{-1} \sum_j 1(P \leq P_j)$$

where the sum is taken over the period $j = \{1, \ldots, T - m\}/\{t_0 - \mathrm{m}, \ldots, t_0 + m - 1\}$. We can compute the p-values of the test statistics for each time period t_0. See Andrews and Kim (2006) for the detailed explanation of the test procedure.

In Table 2 the identified periods of short-run breakdown/instability obtained from the 5% significance level are provided. The identified period in Table 2 is the period in which the p-values are below 5%. The results are obtained for $m = 8$.[1]

The breakdown period for the U.S. data corresponds to the period of monetary policy change in the U.S. from the interest rate pegging policy to the money volume controlling policy. For the Korean data the break-

[1] We also performed the test with $m = 2$ and 4, whose results are similar to those with $m = 8$.

down period contains the period of presidental assassination and subsequent turmoil followed by an important change in economic policy.

V. Analysis of Fractional Integration

An integrated process y_t of order d is defined as

$$(1-L)^d y_t = \psi(L)\varepsilon_t$$

where L is the lag operator and $\psi(L)=\sum_{j=0}^{\infty}\psi_j L^j$ for $\sum_{j=0}^{\infty}|\psi_j|<\infty$. The value of d may or may not be an integer. For an $I(1)$ series $d=1$. If $0<d<1$, then the variable y_t has properties between stationary $I(0)$ and nonstationary $I(1)$ processes. In this case a shock ε_t has impacts on y that survive long after t (long memory), if not forever as for $I(1)$. For more information about the fractional integration, see Robinson (2003), among others. For $-1/2<d<1/2$ the process is a fractionally stationary series, and for $1/2\leq d<1$ it is a fractionally nonstationary series. An advantage of this approach is that the "degree" of stationarity/nonstationarity for a variable can be estimated, whereas the usual unit root tests determine only one of the two "extreme" cases of $I(1)$ and $I(0)$. We apply the method in Phillips (2005) to our data to estimate d.

Table 3 shows the estimation result of the integration coefficient d for the real interest rate.[2] The estimated value of d for the U.S. ($\hat{d}=0.63$) is clearly in the nonstationary region with the 95% confidence interval of d heavily skewed to the nonstationary region. On the other hand, the estimated value of d for Korean data ($\hat{d}=0.51$) is marginally in the nonstationary region with the 95% confidence interval of d extending over both regions of stationarity and nonstationarity almost symmetrically. These estimated results are consistent with the results of ADF test in Table 2.

Table 4 shows the estimation result of d for a smaller sample, where the smaller sample is obtained by removing the observations of the periods of the identified short-run instability (Table 2) from the whole sample. Notice that in this case the estimated values of d are substantially lower for both countries than those in Table 3. Also, notice that the confidence intervals are all heavily skewed to the stationary region ($d<1/2$). Thus, this result provides positive evidence for our earlier conjecture that the Fisher relation holds in the majority of the data

[2] In the estimation the frequency ordinate is set to be $m=n^{0.8}$.

TABLE 3
ESTIMATION RESULTS OF d

	$\hat{d}$	s.d.	95% c.i.
U.S.	0.63	0.13	[0.36, 0.89]
Korea	0.51	0.16	[0.20, 0.81]

TABLE 4
ESTIMATION RESULT OF THE INTEGRATION COEFFICIENT WITH THE OBSERVATIONS OF THE IDENTIFIED INSTABILITY REMOVED

	$\hat{d}$	s.d.	95% c.i.
U.S.	0.12	0.14	[-0.16, 0.39]
Korea	0.00	0.18	[-0.34, 0.35]

period, whereas in certain short period the relation is violated to a substantial degree.

VI. Concluding Remarks

This paper presents new empirical evidence for the Fisher relation. For data from the U.S. and Korea, the Fisher relation holds as a stable relation except for certain relatively short periods of instability. Our result resolves empirical skepticism on the theoretically simple Fisher relation. This result would motivate further investigation of related issues. For example, the analysis of the paper can be extended to data of other countries. Furthermore, there are certain other subjects that are not studied in the paper but are related to the analysis of this paper. An example is the role of the individual variables, the nominal interest rate and the inflation rate, for the Fisher relation. In particular, the relative role of the two variables with respect to the short-run instability of the Fisher relation would be interesting to investigate. In addition, the relationship between the expectation error and the short-run instability of the Fisher relation are also worthwhile subjects for future study.

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References

Andrews, D. W. K., and J. Y. Kim. "Tests for Cointegration Breakdown

over a Short Time Period." *Journal of Business & Economic Statistics* 24 (No. 4 2006): 379-94.

Atkins, F. "Co-integration, Error Correction and the Fisher Effect." *Applied Economics* 21 (No. 12 1989): 1611-20.

Clarida, R. H., and Friedman, B. M. "The Behavior of U.S. Short-Term Interest Rates Since October 1979." *Journal of Finance* 39 (No. 3 1984): 671-82.

Crowder, W. J., and Hoffman, D. L. "The Long-Run Relationship between Nominal Interest Rates and Inflation: The Fisher Equation Revisited." *Journal of Money, Credit and Banking* 28 (1996): 102-18.

Evans, M. D. D., and Lewis, K. K. "Do Expected Shifts in Inflation Affect Estimates of the Long-Run Fisher Relation?" *Journal of Finance* 50 (No. 1 1995): 225-53.

Fama, E. F. "Short-term Interest Rates as Predictors of Inflation." *American Economic Review* 65 (1975): 269-82.

Fama, E. F., and Gibbons, M. R. "Inflation, Real Returns, and Capital Investment." *Journal of Monetary Economics* 9 (No. 3 1982): 297-323.

Garbade, K., and Wachtel, P. "Time Variation in the Relationship between Inflation and Interest Rates." *Journal of Monetary Economics* 4 (No. 4 1978): 755-65.

Huizinga, J., and Mishkin, F. S., "Monetary Policy Regime Shifts and the Unusual Behavior of Real Interest Rates." *Carnegie-Rochester Conference Series on Public Policy* 24 (1986): 231-74.

Kim, J.-Y. "Inference on Segmented Cointegration." *Econometric Theory* 19 (No. 4 2003): 620-39.

Mishkin, F. S. "The Real Interest Rate: An Empirical Investigation." *Carnegie-Rochester Conference Series on Public Policy* 15 (1981): 151-200.

_____. "The Real Interest Rate: A Multi-country Empirical Study." *Canadian Journal of Economics* 17 (No. 2 1984): 283-311.

_____. "Is the Fisher Effect for Real?" *Journal of Monetary Economics* 30 (1992): 195-215.

Nelson, C. R., and Schwert, G. W. "Short-term Interest Rates as Predictors of Inflation: On Testing the Hypothesis that the Real Rate of Interest Is Constant." *American Economic Review* 67 (1977): 478-86.

Phillips, P. C. B. "Econometric Analysis of Fisher's Equation." *American Journal of Economics and Sociology* 64 (No. 1 2005): 125-68.

Robinson, P. M. *Time Series with Long Memory*. Oxford: Oxford University

Press, 2003.

Roley, V. V. The Response of Interest Rates to Money Announcements under Alternative Operating Prosedures and Reserve Requirement Systems. NBER Working Paper No. 1812, 1986.

Rose, A. K. "Is the Real Interest Rate Stable?" *Journal of Finance* 43 (No. 5 1988): 1095-112.

Summers, L. H. The Non-Adjustment of Nominal Interest Rates: A Study of the Fisher Effect. NBER Working Paper No.836, 1982.

Wallace, M. S., and Warner, J. T. "The Fisher Effect and the Term Structure of Interest Rates: Tests of Cointegration." *Review of Economics and Statistics* 75 (1993): 320-4.

Walsh, C. E. "Three Questions Concerning Nominal and Real Interest Rates." *Economic Review-Federal Reserve Bank of San Francisco* 4(1987): 5-19.