The Effects of Heterogeneous Beliefs on a Risky Asset's Price and Trading Volume

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This paper examines the effects of different opinions among investors on a risky asset's price and trading volume in a two-period, two-person general equilibrium setting. A comparative static analysis predicts the following: Assuming that investors differ only in subjective probability beliefs about future security payoffs, an increased dispersion of beliefs will decrease the asset price and increase the trading volume in the empirically plausible range of the relative risk aversion coefficient.

I. Introduction

One of the key assumptions in various asset pricing models of financial economics is the assumption of homogeneous beliefs among individuals about future security payoffs.[1] In this paper, we relax this assumption and examine how different beliefs of economic agents will affect a risky asset's price and trading volume in equilibrium. In asset pricing theory, this heterogeneity issue is considered important on the following grounds. First, if heterogeneous beliefs affect asset prices in a way, the validity of numerous studies based on the homogeneous beliefs may be questioned. This is because in the presence of heterogeneous beliefs,[2] the actual asset price behavior would be different from

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[1]For example, see the standard CAPM by Sharpe, Lintner and Mossin, the consumption CAPM by Breeden, the intertemporal CAPM by Merton, and the representative consumer model by Lucas.

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what these studies predict. Second, as Varian (1987) emphasized, difference in beliefs is relevant for generating trade of risky assets while difference in information is not. The latter fact is due to the theory that differences in information are eliminated eventually if prices are fully revealing.[3]

Several studies have investigated the effects of heterogeneous beliefs in financial markets. Some studies focus on the effect on asset prices and others on the effect on trading volume. As for the latter effect, the literature mostly agrees that increased spread of opinions will cause increased trading among investors. In this respect, this paper will reach the same conclusion with Kim (1983), Karpoff (1986), and Varian (1987).[4] However, the economic motivations of investors for trading a risky asset and thus the trade-generating process will be analyzed in more depth below.

With regard to the effect on asset prices, this paper is especially concerned with the results of Miller (1977) and Varian (1985). Miller asserts that with no short sales allowed, the demand for a risky stock will primarily come from the most optimistic investors. Thus, with the supply of the security fixed, the higher the divergence of opinions concerning the stock's future payoffs (with the cross-sectional average of opinions held constant), the higher will be the market clearing price. His argument appears to make sense at first, but he does not have a formal model.

Contrary to Miller's assertion, Varian (1985) shows that under empirically plausible assumptions on the investors' utility function, increased disagreement among agents is associated with lower asset prices. He analyzes the problem in a complete market in the Arrow-Debreu sense, and his approach is a comparative asset pricing.

Motivated by the confliction results of the two studies, this paper analyzes the problem in an alternative framework. As contrasted with Varian (1985), we consider the issue in an incomplete market, and perform a comparative static analysis rather than a comparative asset pricing. The structure of the economy in this paper is a variant of the general equilibrium setting developed by Lucas (1978).

Recently, Abel (1989) used the Lucas setting to study asset pricing

[2]In this paper, we do not address why people have different beliefs or opinions. Rather, we assume that they agree to disagree.

[3]See Grossman (1976).

[4]One exception to this line is the result by Pfleiderer (1984), who reports that expected volume is a decreasing function of the dispersion of expectations.

under heterogeneous expectations. He attempts to partially resolve the equity premium puzzle of Mehra and Prescott by appealing to heterogeneous expectations. While Abel's model and the model presented below have a similar setup, they differ in the following respects. First, Abel's results depend critically on the assumption that the riskless rate of return is determined endogenously in the loan market. By contrast, we show below that the heterogeneity of beliefs affects a risky asset's price even if there is no loan market. Neither lending nor borrowing is allowed by assumption in this paper. Second, while Abel relies on the assumption that investors have Constant Absolute Risk Aversion (CARA), we assume Constant Relative Risk Aversion (CRRA) which is known as being plausible empirically.

The following results are derived in this paper. First, when two investors, motivated by different beliefs, trade a risky asset, the relatively more optimistic person will become the seller (buyer) if the relative risk aversion coefficient is greater (less) than unity. Second, given that initial endowments and preferences of agents are identical, increase in the degree of disagreement between consumers will decrease the equilibrium asset price and increase the equilibrium trading volume in the likely range of the CRRA coefficient.

The remainder of this paper proceeds as follows. In section II, an exchange economy with two heterogeneous consumers is introduced, and its equilibrium is defined. Section III analyzes the effects of divergence of beliefs and obtains the results mentioned above. Concluding remarks are provided in section IV.

II. The Model

Consider a two-period (t = 0, 1) version of Lucas (1978) economy in which one capital unit produces y_t units of a perishable consumption good (numeraire good) each period.[5] The ownership of the capital is represented by a divisible share of stock. The consumption good and the ownership are traded in a stock market. Let p denote an ex-dividend per share price which is competitively determined in the stock market in period 0. Suppose that there are two economic agents (consumers A and B) who are equally endowed (with 1/2 share) and have identical preferences. During the first period, given the dividend income ($\frac{1}{2}y_0$), each consumer allocates it between current consumption, C_0, and

[5]The capital unit is assumed to be valueless after the second period.

investment in the risky stock for future consumption, $\tilde{C}_1$. At the time of consumption decision (t = 0), the current total output, (y_0) is known, but the amount of future output ($\tilde{y}_1$) is uncertain. Each agent believes that $\tilde{y}_1$ is distributed as follows:

States (s)	Output (y_1)	Probability Beliefs Consumer A	Probability Beliefs Consumer B
Good (g)	y_g	π_g^A	π_g^B
Bad (b)	y_b	$1-\pi_g^A$	$1-\pi_g^B$

where $\pi_g^A \neq \pi_g^B$. Without loss of generality, it is assumed that $y_b < y_g$. Hence the two consumers are identical in every respect except for the subjective probability beliefs about future asset payoffs.

In this environment, each consumer (i = A, B) wishes to maximize an objective function

$$E_i[\sum_{t=0}^{1}\beta^t u(C_t^i)] \equiv u(C_0^i)+\beta E_i u(\tilde{C}_1^i) \tag{1}$$

subject to budget constraints

$$C_0^i \leq \frac{1}{2}y_0 - pq^i \tag{2}$$

$$\tilde{C}_1^i \leq (\frac{1}{2}+q^i)\tilde{y}_1 \tag{3}$$

$$C_0^i \geq 0, \ \ \tilde{C}_1^i \geq 0 \qquad (i = A, \ B), \tag{4}$$

where E_i is an expectation operator, u: $\mathbf{R}^+ \rightarrow \mathbf{R}^+$ is a von Neumann-Morgenstern utility function and is assumed to exhibit Constant Relative Risk Aversion (CRRA) of the form $u(C) = \{1/(1-\gamma)\}C^{1-\gamma}$ with $\gamma >$ 0 and $\gamma \neq 1$, q^i is the amount of the share to be bought, β is a discount factor ($0 < \beta < 1$).

With budget constraints (2) and (3) substituted into (1), each individual's decision problem is written as

$$\underset{q^i}{\text{Max}}\ [u(\frac{1}{2}y_0 - pq^i)+\beta E_i u((\frac{1}{2}+q^i)\tilde{y}_1)], \quad (i = A, \ B). \tag{5}$$

The optimality condition for each consumer is

$$u'(C_0^i)p - \beta E_i[u'(\tilde{C}_1^i)\tilde{y}_1] = 0, \quad (i = A, \ B). \tag{6}$$

As is well-known, this first-order condition says that the utility gain at time 0 must be equal to the discounted expected value of utility loss at time 1.

Once the optimal decision rule is determined from condition (6) at an individual level, the rules must satisfy the following market clearing condition to reach equilibrium:

$$q^A + q^B = 0. \tag{7}$$

That is, $q^A = -q^B$, saying that consumer B must be the seller (buyer) if consumer A is the buyer (seller). Some notational changes are made here for convenience in later use. Let $q^A \equiv q$, then $q^B = -q^A \equiv -q$.

Evaluating each optimality condition of (6) in equilibrium using these notations and the CRRA utility, we may write each condition as follows:

$$(\frac{1}{2}y_0 - pq)^{-\gamma} p = \beta E_A[\{(\frac{1}{2}+q)\tilde{y}_1\}^{-\gamma}\tilde{y}_1] \tag{8}$$

$$(\frac{1}{2}y_0 + pq)^{-\gamma} p = \beta E_B[\{(\frac{1}{2}-q)\tilde{y}_1\}^{-\gamma}\tilde{y}_1]. \tag{9}$$

Solving equations (8) and (9) for decision variables, and expanding the expectation operators with respect to subjective probability beliefs, we derive the following demand (or supply) function for the risky asset of each consumer.

$$q = \frac{1}{2}\left[\frac{y_0 - (p/\beta M_A)^{1/\gamma}}{p + (p/\beta M_A)^{1/\gamma}}\right] \tag{10}$$

$$-q = \frac{1}{2}\left[\frac{y_0 - (p/\beta M_B)^{1/\gamma}}{p + (p/\beta M_B)^{1/\gamma}}\right] \tag{11}$$

$$\text{where } M_A \equiv \pi_g^A y_g^{1-\gamma} + (1-\pi_g^A)y_b^{1-\gamma}$$

$$M_B \equiv \pi_g^B y_g^{1-\gamma} + (1-\pi_g^B)y_b^{1-\gamma}.$$

A couple of notes deserve to be made here. First, given the asset price, each demand is proportional to the initial endowment (1/2), which reflects the property of the CRRA utility that agents consume a fixed proportion of their initial wealth and invest the rest for future consumptions. Second, the supply curve is negatively sloped if $0 < \gamma < 1$, but backward-bending if $\gamma > 1$ [see Figure 1 below]. The reason for being backward-bending in the case $\gamma > 1$ is that beyond some price level, income effect is large enough to dominate substitution effect.

From equations (10) and (11), we can determine the market clearing price (p^*) and the amount of share to be bought by consumer A (q^*),[6]

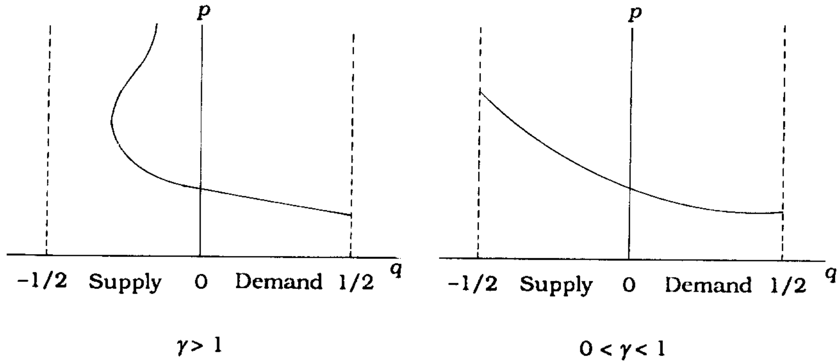

FIGURE 1
SHAPES OF SUPPLY CURVES DEPENDING ON THE DEGREE OF THE RISK AVERSION COEFFICIENT, γ.

which define the equilibrium of the economy. Note that if q^* is positive (negative), consumer A is the buyer (seller) of the risky asset. We refer to the absolute value of q^* ($|q^*|$) as trading volume.

III. The Effects of Heterogeneous Beliefs

We now turn to the main concern of this paper. In equations (10) and (11), we are going to examine how p^* and $|q^*|$ depend on the differences of opinion between the two consumers. In order to measure the degree of difference, let us assume that consumer A is relatively more optimistic than B ($\pi_g^A > \pi_g^B$). This does not lose generality since the two consumers are identical otherwise. Thus we express agents' beliefs as follows:

$$\pi_g^A + \pi_g^B = \alpha \quad (0 < \alpha < 2,\ \text{constant}) \tag{12}$$

$$\pi_g^A - \pi_g^B = h \quad (0 < h < 1). \tag{13}$$

These expressions have the following implications. Given α, the cross-sectional mean of beliefs is held constant ($\alpha/2$) and the cross-sectional variance of beliefs increases as h increases. In other words, with an increase in h, one cross-sectional distribution of beliefs for a state is a distribution for the state.[7] Thus, varying h allows us to measure the

[6]Throughout the remainder of this paper, the ' * ' superscript indicates an optimal solution.

[7]Varian (1985) also imposes a similar restriction in order to apply the second-order stochastic dominance principle in the Rothschild and Stiglitz sense.

degrees of heterogeneity in beliefs.

For simplicity, we also assume the following on the amount of current output:

$$y_0 = 1. \tag{14}$$

This assumption is justified on the ground that q^* is homogeneous of degree 0 and p^* is homogeneous of degree 1 in y_0, y_g, and y_b.[8] Thus, q^* is not affected by, but p^* is unique up to, the scale factor of y_0, y_g, and y_b. In consequence of (14), y_g and y_b may be thought of as the (gross) growth rates of output in each state.

With (12)-(14), we may rewrite (10) and (11) as follows,respectively.

$$f^1(p,q;\ h) \equiv \frac{1}{2}\left[\frac{1-(p/\beta M_A)^{1/\gamma}}{p+(p/\beta M_A)^{1/\gamma}}\right] - q = 0 \tag{15}$$

$$f^2(p,q:\ h) \equiv \frac{1}{2}\left[\frac{(p/\beta M_B)^{1/\gamma}-1}{p+(p/\beta M_B)^{1/\gamma}}\right] - q = 0 \tag{16}$$

$$\text{where}\quad M_A \equiv (1/2)[(\alpha+h)y_g^{1-\gamma} + (2-\alpha-h)y_b^{1-\gamma}]$$

$$M_B \equiv (1/2)[(\alpha-h)y_g^{1-\gamma} - (2-\alpha+h)y_b^{1-\gamma}].$$

These two equations form a simultaneous equations system where p and q are endogenous variables and h is the parameter of our main concern. It is not difficult to show that the above system is stable, and that solutions exist and are unique over relevant ranges of variables.[9] Then, the following is found.

Proposition 1

The relatively more optimistic person, consumer A, becomes the seller (buyer) of the asset in equilibrium if the CRRA coefficient, γ, is greater (less) than unity.

Proof: Observe first that $1/M_A^{1/\gamma} > (<)\ 1/M_B^{1/\gamma}$ if γ >(<) 1.[10] From (15) and (16) the following is true.

[8]Multiplying by a constant both numerator and denominator of equation (10) or (11) will prove this.

[9]Using the global univalence theorem, we only have to prove that the Jacobian determinant, $|J|$, is positive, where

$$|J| = \begin{vmatrix} \partial f^1/\partial p^* & \partial f^1/\partial q^* \\ \partial f^2/\partial p^* & \partial f^2/\partial q^* \end{vmatrix}.$$

[10]This can be shown as follows: $M_A - M_B = h(y_g^{1-\gamma} - y_b^{1-\gamma}) < (>)\ 0$ since $h > 0$ and $y_g^{1-\gamma} < (>) y_b^{1-\gamma}$ if $\gamma > (<)\ 1$. Therefore, $1/M_A^{1/\gamma} > (<) 1/M_B^{1/\gamma}$ if $\gamma > (<)\ 1$.

$$\left[\frac{1-(p/\beta M_A)^{1/\gamma}}{p+(p/\beta M_A)^{1/\gamma}}\right]+\left[\frac{1-(p/\beta M_B)^{1/\gamma}}{p+(p/\beta M_B)^{1/\gamma}}\right]=0 \tag{17}$$

Since both denominator terms are positive, one of the numerators must be positive and the other must be negative. Since $1/M_A^{1/\gamma} > (<)$ $1/M_B^{1/\gamma}$, $1 - (p/\beta M_A)^{1/\gamma} < (>)\ 0$ and $1 - (p/\beta M_B)^{1/\gamma} > (<)\ 0$ if $\gamma > (<)\ 1$. Therefore, $q^* < (>)\ 0$ if $\gamma > (<)\ 1$.

Q.E.D.

An economic reasoning for the above result is as follows: Since M_A and M_B are expected values of marginal utility times total future output, they represent social benefits (in terms of utility) from future output expected by consumers A and B, respectively. As M_A is less (greater) than M_B for $\gamma > (<)\ 1$, consumer A is willing to sell (buy) part of his (the other agent's) claim to future consumption in order to increase his current consumption. An alternative explanation is: When γ is greater (less) than unity, the dominant income (substitution) effect makes consumer A have greater (less) incentive to increase (decrease) current consumption and thus to sell (buy) the asset.

With Proposition 1, the following comparative static results can be derived.

Proposition 2

As the probability beliefs of the two agents diverge [As h increases], the asset price (p^*) decreases for all $\gamma > 1$ and some $0 < \gamma < 1$, and the trading volume ($|q^*|$) increases for all $\gamma > 0$.

Proof: We need to check the signs of $\partial p^*/\partial h$ and $\partial q^*/\partial h$ in equations (15) and (16). Applying the method of comparative statics,[11] we obtain the following relationships.

$$\frac{\partial f^1}{\partial p^*}\times\frac{\partial p^*}{\partial h}+\frac{\partial f^1}{\partial q^*}\times\frac{\partial q^*}{\partial h}=-\frac{\partial f^1}{\partial h} \tag{18}$$

$$\frac{\partial f^2}{\partial p^*}\times\frac{\partial p^*}{\partial h}+\frac{\partial f^2}{\partial q^*}\times\frac{\partial q^*}{\partial h}=-\frac{\partial f^2}{\partial h} \tag{19}$$

Solutions to these simultaneous equations are [by Cramer's rule]

$$\frac{\partial p^*}{\partial h}=\frac{-\dfrac{\partial f^2}{\partial h}\dfrac{\partial f^1}{\partial q^*}+\dfrac{\partial f^1}{\partial h}\dfrac{\partial f^2}{\partial q^*}}{\dfrac{\partial f^2}{\partial p^*}\dfrac{\partial f^1}{\partial q^*}-\dfrac{\partial f^1}{\partial p^*}\dfrac{\partial f^2}{\partial q^*}} \tag{20}$$

[11]See Chapter 2 in Samuelson (1983).

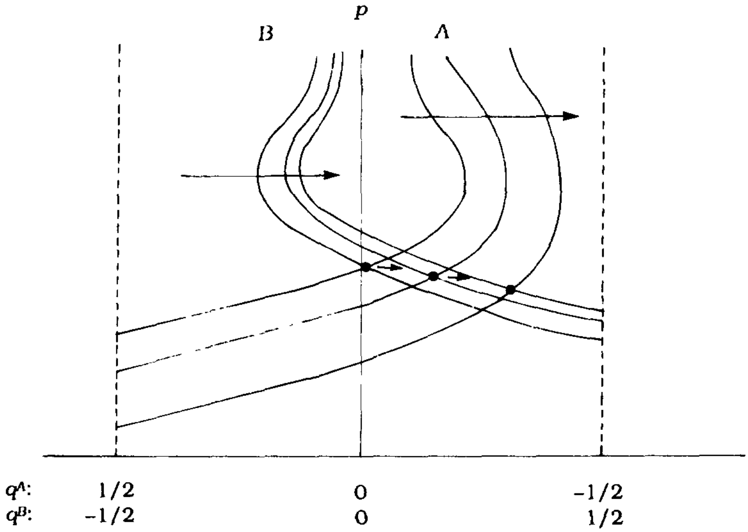

Figure 2
Changes in Equilibrium Price and Trading Volume with Increased Heterogeneity of Beliefs (in the Case of $\gamma > 1$).

$$\frac{\partial q^*}{\partial h} = \frac{-\dfrac{\partial f^1}{\partial h}\dfrac{\partial f^2}{\partial p^*} + \dfrac{\partial f^2}{\partial h}\dfrac{\partial f^1}{\partial p^*}}{\dfrac{\partial f^2}{\partial p^*}\dfrac{\partial f^1}{\partial q^*} - \dfrac{\partial f^1}{\partial p^*}\dfrac{\partial f^2}{\partial q^*}}. \tag{21}$$

In the Appendix, the following results are derived:

i) $\partial p^*/\partial h < 0$ for all $\gamma > 1$ and some $0 < \gamma < 1$
ii) $\partial q^*/\partial h < (>) 0$ for $\gamma > 1$ $(0 < \gamma < 1)$.

These results prove Proposition 2. Notice that for $\gamma > 1$, decreasing q^* implies increasing trading volume since q^* is negative.

A graphical depiction of Proposition 2 (for $\gamma > 1$ case) is as follows [see Figure 2 above]. While both supply curve and the demand curve shift right-ward in response to an increased spread of beliefs (the signs of $\partial f^1 / \partial h$ and $\partial f^2 / \partial h$ are both negative), the degree of shift of the supply curve is greater than that of the demand curve ($|\partial f^1 / \partial h|$ is greater than $|\partial f^2 / \partial h|$).[12] This will lead to the price and volume results above.

[12]For some values of $0 < \gamma < 1$, this statement is still true. For other values of $0 < \gamma < 1$, however, the first part of the statement is true but the second part is

TABLE 1
NUMERICAL EXAMPLES

		$\gamma = 1/2$		$\gamma = 1$		$\gamma = 2$	
π_g^A	π_g^B	q^*	p^*	q^*	p^*	q^*	p^*
0.9	0.1	0.0406	0.9917	0	1	–0.0199	1.0282
0.8	0.2	0.0304	0.9931	0	1	–0.0148	1.0341
0.7	0.3	0.0203	0.9941	0	1	–0.0098	1.0383
0.6	0.4	0.0101	0.9947	0	1	–0.0049	1.0408
0.5	0.5	0.0000	0.9949	0	1	–0.0000	1.0417
		$\gamma = 3$		$\gamma = 4$		$\gamma = 5$	
π_g^A	π_g^B	q^*	p^*	q^*	p^*	q^*	p^*
0.9	0.1	–0.0255	1.0731	–0.0272	1.1383	–0.0268	1.2295
0.8	0.2	–0.0187	1.0975	–0.0192	1.1945	–0.0182	1.3350
0.7	0.3	–0.0122	1.1148	–0.0123	1.2349	–0.0114	1.4065
0.6	0.4	–0.0060	1.1250	–0.0060	1.2582	–0.0055	1.4482
0.5	0.5	–0.000	1.1285	–0.0000	1.2659	–0.0000	1.4618

Note: Equilibrium values of a risky asset's price and trading volume with changes in heterogeneity of beliefs ($h \equiv \pi_g^A - \pi_g^B$), assuming the following parameter values in equations (10) and (11): $\beta = 1$, $y_0 = 1$, $y_g = 1.2$, $y_b = 0.8$, $\alpha (\equiv \pi_g^A + \pi_g^B) = 1$.

Now let us turn to the two special cases that we have not considered yet: i) The log utility case ($\gamma = 1$) [$u(C) \equiv \ln C$] and ii) risk neutrality case ($\gamma = 0$). In the case $\gamma = 1$, it follows from (10) and (11) that $p^* = \beta y_0$ and $q^* = 0$. Interestingly, consumers do not trade each other in this case, and belief parameters do not play any role in determining the equilibrium price. These results are due to the property of log utility that the substitution effect and the income effect exactly cancel out each other. In the case $\gamma = 0$, one is willing to sell the asset if the price is higher than his (subjective) expectation about future payoffs. The other one is willing to buy it if the price is lower than his expectation. Therefore, trading will continue to occur until agents sell out their initially endowed shares. Thus, equilibrium values are such that $q^* = 1/2$ and p^* is any value between the two expectations. Hence, the degree of difference in beliefs is irrelevant for determining the volume of trade. Only a small difference may generate a large volume of trade.

To illustrate the results obtained thus far, we provide numerical

ambiguous. This is why the price effect is ambiguous in some range of $0 < \gamma < 1$.

examples in the Table 1 that were generated from a computer program.

IV. Concluding Remarks

This paper investigates how disagreement among investors affects a risky asset's price and trading volume. Assuming that agents have an intertemporally stationary time-additive expected utility, our analysis shows that increased heterogeneity of beliefs will decrease the asset price and increase the volume of trade in the empirically plausible range of the risk aversion coefficient.[13] While these results are obtained in a two-agent, two-state framework, the fundamental intuitions should be robust in a world with many agents and many states.

Since the model in this paper is contrasted with Varian's model (1985) in many respects, we conclude the paper by comparing the two studies. First, this study deals with an intertemporal utility maximization problem as compared to the static utility maximization problem in Varian's study. Thus, the preference parameter here (denoted as γ) governs both risk aversion and intertemporal substitution. Its role as a substitution parameter is well-demonstrated in Proposition 1, in the sense that the magnitude of γ determines whether income (or substitution) effect is dominant. Second, as Varian uses weighted probability beliefs with the weights being the inverses of marginal utility of income, he does not investigate the effect due purely to different opinions. The wealth effect is compounded in his model. This may be one potential reason why the results of the two studies are rather different. While Varian's results say that the sign of the price effect is negative or positive depending on whether the CRRA coefficient is greater or less than unity, this paper shows that the price effect may be negative even with the coefficient less than unity. The cause of this discrepancy will be clear if we eliminate the difference of the two models in market completeness. Introducing another risky asset in our model will enable us to examine whether completing the market affects asset pricing under heterogeneous beliefs. This is left as a future research subject.

[13]The empirical study by Blume and Friend (1975) concludes that the Constant Relative Risk Aversion (CRRA) coefficient (γ) is at least two. Also see Grossman and Shiller (1981).

Appendix

In this appendix, we check the signs of $\partial p^*/\partial h$ and $\partial q^*/h$ in equations (20) and (21). Denoting the right-hand sides of (20) and (21) by N_1/D and N_2/D respectively, we determine the signs of D, N_1, and N_2 with each derivative term calculated. Using the result in footnote 9, we easily obtain the sign of D, which turns out to be negative. Then, we need the following lemma to check the signs of N_1 and N_2.

Lemma

Let $$F(t) \equiv \frac{t^{(-1-\gamma)/\gamma}}{\{p+(p/\beta t)^{1/\gamma}\}^2} \quad (t > 0).$$

Then, $F'(t) < 0$ for all $\gamma > 1$ and some $0 < \gamma < 1$, where F' denotes the first derivative of F.

Taking the derivative of F with respect to t will prove this lemma. We then check the signs of N_1 and N_2 below.

(1) *The sign of* N_1: N_1 is calculated as

$$N_1 = (1/4\gamma)(p/\beta)^{1/\gamma}(p+1)(y_g^{1-\gamma} - y_b^{1-\gamma})[F(M_B) - F(M_A)],$$

where function F is defined as in Lemma, and M_i ($i = A$, B) as in equations (15) and (16) in the text. For $\gamma > 1$, the bracketed term is negative by Lemma (using the fact $M_A < M_B$), and $(y_g^{1-\gamma} - y_b^{1-\gamma})$ is negative. Thus N_1 is positive. For some values $0 < \gamma < 1$, the bracketed term is positive and $(y_g^{1-\gamma} - y_b^{1-\gamma})$ is positive. N_1 is still positive.

(2) The sign of N_2: N_2 is calculated as

$$N_2 = (1/4\gamma)(p/\beta)^{1/\gamma}(p+1)(y_b^{1-\gamma} - y_g^{1-\gamma})[F(M_A)\times B_B + F(M_B)\times B_A],$$

where $$B_i \equiv \frac{\{1-(p/\beta M_i)^{1/\gamma}\}+(1/\gamma)(p/\beta M_i)^{1/\gamma}\{1+(1/p)\}}{D_i^2}$$ and

$$D_i \equiv p+(p/\beta M_i)^{1/\gamma} \quad \text{for} \quad i = A, B.$$

For $0 < g < 1$, if we rewrite the numerator of B_i as $[\{(1/\gamma)-1\}(p/\beta M_i)^{1/\gamma} +$ positive terms$]$, all the terms in the bracket are shown to be positive. Since $(y_g^{1-\gamma} - y_b^{1-\gamma})$ is negative, N_2 is negative.

For $\gamma > 1$, since $(y_b^{1-\gamma} - y_g^{1-\gamma}) > 0$, $B_B > 0$, and $F(M_A) > F(M_B) > 0$ by Lemma, the following is true.

$$N_2 = (1/4\gamma)(p/\beta)^{1/\gamma}(p+1)(y_b^{1-\gamma} - y_g^{1-\gamma})F(M_B)[B_B + B_A].$$

Rewriting equation (17) in the text as follows,

$$\frac{1-(p/\beta M_B)^{1/\gamma}}{D_B} = \frac{(p/\beta M_A)^{1/\gamma}-1}{D_A},$$

we can show

$$B_B + B_A = (1/2D_A^2 D_B^2)[D_A(D_A - D_B)\{1-(p/\beta M_B)^{1/\gamma}\} + \text{positive terms}].$$

Since $D_A > D_B$ and $1 - (p/\beta M_B)^{1/\gamma} > 0$ for $\gamma > 1$, $[B_B + B_A] > 0$. Hence, N_2 is positive.

From combining all the results in this Appendix, Proposition 2 follows.

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