Interdependent Value Auctions with an Insider Bidder

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We study the efficiency of standard auctions with interdependent values in which one of two bidders is perfectly informed of his value while the other is partially informed. The second-price auction, as well as English auction, has a unique ex-post equilibrium that yields efficient allocation. By contrast, the first-price auction has no efficient equilibrium.

Keywords: Interdependent value, Insider, Second-price auction, First-price auction

JEL Classification: C92, D44, D82

I. Introduction

Most auction literature assumes that bidders are ex ante homogeneous in terms of the amount of the information they hold about the auctioned object. More specifically, each bidder is assumed to hold a single-dimensional information (or signal) about the value of the object. In reality, however, there are many instances in which bidders hold heterogeneous information in terms of its informativeness of the object value. For instance, in a spectrum auction, an incumbent company that has been doing business for a long time should have an advantage in the estimation of the object value compared with newcomers. In addition, in an auction for selling a tract in the outer continental shelf (so-called OCS auction), a company would have an informational advantage

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over others if it owns and has drilled a neighboring tract.[1] The bidders who have superior information as in the above examples can be considered insiders in the auction.

The current paper studies the efficiency implication of an insider bidder in standard auctions with two bidders, in which one is an insider and the other is an outsider. Following the standard approach, the value interdependence is modeled by assuming that the two bidders' values are determined by two-dimensional signals. The outsider bidder is only informed of one of the two signals, so he is partially informed of his value. The paper departs from the existing literature by assuming that the insider is fully informed of his value. We ask whether the standard auctions — first-price, second-price, and English auctions — have an efficient equilibrium (*i.e.*, an equilibrium in which the object is allocated to whomever has the higher value).[2]

The two auction formats, first-price and second-price auctions, exhibit a contrasting efficiency performance: the unique undominated Nash equilibrium of the second-price auction yields efficient allocation, whereas the first-price auction does not admit any efficient equilibrium. The difference between the two formats can be explained using the fact that the second-price auction has an ex-post, efficient equilibrium when two bidders exist. The ex-post equilibrium means that neither bidder has an incentive to submit a bid different from the equilibrium bid, even after he learns all the information and precisely knows his value. Consider first the standard setup (Milgrom, and Weber 1982) in which both bidders are outsiders in the sense of holding single-dimensional signals. In this setup, an efficient ex-post equilibrium exists (Maskin 1992). Suppose that one bidder, say bidder 1, has a lower value, so bidder 2 is a winner in the equilibrium. By the property of the ex-post equilibrium, the equilibrium bid of bidder 2 must be higher than the value of bidder 1; otherwise, bidder 1 would have an incentive to deviate and win the object if he precisely knows his value. Consider the case in which bidder 1 becomes an insider and employs the dominant strategy of bidding his value while bidder 2 remains an outsider. Despite this change, bidder 2 has no incentive to revise his bid from the standard case because it is still higher than bidder 1's new bid (which is bidder 1's value and lower

[1] Refer to Hendricks *et al.* (1994).

[2] As will be explained later, when there are two bidders, the second-price and English auctions are strategically equivalent. Thus, our results with the second-price also apply to the English auction.

than bidder 2's value) and he is paying less than his value and cannot affect the price he pays. By contrast, the first-price auction does not admit any ex-post equilibrium, because a bidder can reduce the price he pays as a winner by decreasing his bid.[3] Thus, once a bidder becomes an insider, he will reflect extra information when deciding how much to shade his bid. As a consequence, he may end up bidding a different amount than the outsider who has the same value, which will cause an inefficient allocation.

The first-price auction with asymmetric bidders has long been the subject of analysis in the existing literature. Lebrun (1999) has established the uniqueness of the equilibrium. Maskin, and Riley (2000a) have extended the existence to a more general setup. Maskin and Riley (2000b) compared the first-price and second-price auctions in terms of their revenues. These studies consider the private value setup in which bidders are asymmetric in their value distributions. In the private value setup, the second-price auction yields the efficient allocation but the first-price auction does not. The current paper shows that this comparison continues to hold in the interdependent value setup in which bidders are asymmetric in the amount of information about the object value. Engelbreche-Wiggans *et al.* (1982), Campbell, and Levin (2000), and Kim (2008) have also studied the first-price auction with an insider bidder. They assume that bidders have a common value, so there is no efficiency issue; instead, they focus on the characterization of equilibrium and its revenue implication.

II. Model

A seller has an object to sell to one of two bidders, bidder 1 and bidder 2. The value of the object to bidder $i=1, 2$ is a function of two-dimensional signal profile $s=(s_1, s_2)\in[0, 1]^2$ and denoted by $v_i(s)$.[4] We follow the convention of calling s_i the bidder i's signal. The signal profile (s_1, s_2) is distributed according to F: $[0, 1]^2\rightarrow[0, 1]$ with density f. The value functions are assumed to be continuously differentiable and satisfy $v_i(s)\geq 0,\ \forall s\in[0, 1]^2$ and

[3] Allocational inefficiency caused by asymmetric information has also been studied by Chang (1990).

[4] Throughout the paper, we use the capital letter to denote a random variable and the small letter to denote its realization.

$$\frac{\partial v_i}{\partial s_i}(s) > \frac{\partial v_j}{\partial s_i}(s) \geq 0 \text{ for all } s \text{ and } i \neq j. \quad (1)$$

That is, each bidder's signal affects his own value more than the other bidder's value.[5] Note that two bidders are allowed to have different values, although the value of each bidder depends on the entire signal profile, which makes the efficient allocation a nontrivial problem. It is different from the common value setup in which the efficiency issue is absent.

Let us turn to modeling of an insider bidder. In most existing literature, bidders are rather homogeneous in the sense that every bidder observes a one-dimensional signal, which will henceforth be referred to as the standard (information) setup. We depart from this literature by assuming that bidder 1 is an insider who precisely knows the realization of his value (*i.e.*, $v_1(s)$), whereas bidder 2 is an outsider who only knows his signal s_2. The information structure described so far is common knowledge among bidders. In particular, bidder 2 knows that bidder 1 is an insider.

We consider two auction formats, namely, first-price and second-price auctions. In both auctions, each bidder submits a sealed bid and the highest bidder wins the object. The price the winner pays is the first highest bid (*i.e.*, his own bid) in the first-price auction, and the second highest bid in the second-price auction. Ties are broken randomly. The analysis of the second-price auction applies to English auction without any change. To explain, consider an English auction of the Japanese format, and its rule is described as follows: there is a price clock that continuously rises starting from zero; bidders gradually drop out of the auction until only one bidder remains and is awarded the object at the last drop-out price. Given that the price and allocation rules are the same across the two auction formats, the difference only comes from the fact that in an English auction, a bidder can observe the prices at which others have dropped out and make inferences about their signals, but he is unable to do so in the second-price auction. However, with two bidders, an English auction ends as soon as one bidder drops out, so such information is unavailable.

In the second-price auction, the insider, bidder 1, is assumed to

[5] Maskin (1982) has shown that without this condition, implementing the efficient allocation is impossible via any mechanism. One can extend his argument to establish the same result in our setup.

employ a (weak) dominant strategy of bidding his value. Given this, the analysis of the second-price auction proceeds without further specifying the insider's information other than that he knows his value precisely. By contrast, the analysis of the first-price auction does require specifying what information precisely the insider has, because a bidder in the first-price auction wants to shade bid and the ability to do so is affected by his knowledge about the rival's signal (or rival's bid).[6] In this paper, we consider two cases:

***Case* 1:** Bidder 1, who is an insider, knows the realized value of $v_1(s)$, and nothing else;
***Case* 2:** Bidder 1 knows the entire signal profile (s_1, s_2).

Although the two cases are identical in terms of the bidder 1's knowledge about his value, he has more refined knowledge about his rival's signal in Case 2 than in Case 1. Note that bidder 1's information is multidimensional in Case 2.

III. Analysis

We first analyze the second-price auction by characterizing a unique undominated Bayesian Nash equilibrium. To do so, let us define the following bidding strategy:

$$\beta_i(s_i) \equiv \begin{cases} \textit{some } \; b \in [v_i(s_i, 0), v_j(s_i, 0)] & \text{if } \; v_i(s_i, 0) < v_j(s_i, 0) \\ \textit{some } \; b \in [v_j(s_i, 1), v_i(s_i, 1)] & \text{if } \; v_i(s_i, 1) > v_j(s_i, 1) \\ \max\{v_i(s_i, s_j) : v_i(s_i, s_j) \geq v_j(s_i, s_j), s_j \in [0, 1]\} & \text{if otherwise.} \end{cases} \tag{2}$$

Maskin (1982) shows that this bidding strategy constitutes an ex-post equilibrium of the second-price auction in the standard setup in which each bidder i only knows s_i.[7] The following proposition establishes that the bidding strategy in Equation (2) continues to constitute an equilibrium strategy for bidder 2 as bidder 1 becomes an insider and employs the (weak) dominant strategy of bidding his own value $v_1(s)$.

6 For instance, Kim, and Che (2004) show how the information about rivals' types can affect the performance of the first-price auction in the private value setup.

7 The ex-post equilibrium is a Bayesian Nash equilibrium in which each player has no incentive to change his action even after he learns all players' information.

***Proposition* 1:** In any undominated Bayesian Nash equilibrium of the second-price (or English) auction, bidder 1 bids his value $v_1(s)$ while bidder 2 follows the bidding strategy β_2. This strategy profile also constitutes an ex-post equilibrium. The object is efficiently allocated in this equilibrium.

Proof. Obviously, the only undominated strategy of bidder 1 is to bid $v_1(s)$ for every realization. It is clearly a dominated strategy for bidder 2 with signal s_2 to bid more than the highest possible value $v_2(1, s_2)$ or less than the lowest possible value $v_2(0, s_2)$. We show that the only optimal strategy for bidder 2 is to bid $\beta_2(s_2)$ if his signal is s_2. We prove this by considering three cases separately.

(i) Consider first a signal $s_2 \in [0, 1]$ such that $v_1(0, s_2) > v_2(0, s_2)$. Then, the single crossing property as in Equation (1) implies that $v_1(s_1, s_2) > v_2(s_1, s_2)$, $\forall s_1 \in [0, 1]$. As bidder 1 bids $v_1(s)$, bidder 2 would incur a loss by winning and paying $v_1(s)$. To avoid this loss, bidder 2 must bid no more than $v_1(0, s_2)$ and lose to bidder 2. Thus, he must bid $b \in [v_2(0, s_2), v_1(0, s_2)]$ in any undominated equilibrium. This yields the efficient allocation as bidder 1 has a higher value irrespective of s_1.

(ii) For signal s_2 such that $v_1(1, s_2) < v_2(1, s_2)$, a similar argument as in the case (i) can be used to show that bidder 2 must bid $b \in [v_1(1, s_2), v_2(1, s_2)]$ in any undominated equilibrium. In this case, bidder 2 wins the object irrespective of s_1, so the allocation is efficient.

(iii) Consider a signal $s_2 \in [0, 1]$ such that neither case (i) nor case (ii) holds. By the single crossing property, there is a unique $\alpha \in [0, 1]$ such that $v_1(\alpha, s_2) = v_2(\alpha, s_2)$. Given that bidder 1 is bidding his value, it is clearly suboptimal for bidder 2 to bid any $b < v_1(0, s_2)$ (and lose to bidder 1 irrespective of s_1) or to bid any $b > v_1(1, s_2)$ (and win irrespective of s_1). Thus, any optimal bid b must lie in $[v_1(0, s_2), v_1(1, s_2)]$, so there exists $\phi_1(b, s_2) \in [0, 1]$ such that $v_1(\phi_1(b, s_2), s_2) = b$. Bidder 2's expected payoff from bidding such b is

$$\int_0^{\phi_1(b,s_2)} [v_2(s_1, s_2) - v_1(s_1, s_2)] f_{S_1|S_2}(s_1 \mid s_2) ds_1,$$

where $f_{S_1|S_2}(\cdot|\cdot)$ is the density of S_1 conditional on the realization of S_2. The integrand is positive if and only if $s_1 < \alpha$, so the above expression is maximized by setting $b = \beta_2(s_2) = v_1(\alpha, s_2) = v_2(\alpha, s_2)$. Hence, bidder 2 wins if and only if $s_1 < \alpha$ or $v_1(s_1, s_2) < v_2(s_1, s_2)$ because of the single crossing condition, which means that the allocation is efficient.

In all three cases, bidder 1 is bidding more (less) than bidder 2's value whenever he is winning (losing), which implies that bidder 2 has no incentive to deviate from β_2 even when he learns s_1. Thus, the specified strategy profile also constitutes an ex-post equilibrium.

Q.E.D.

Note that the uniqueness and efficiency of the equilibrium described in Proposition 1 are in contrast to the standard information setup in which a plethora of (undominated) inefficient equilibria for the second-price auction exists (refer to Bikhchandani, and Riley (1991) or Chung, and Ely (2001) for this multiplicity).

The presence of an insider has a revenue implication that the seller's revenue (weakly) increases as bidder 1 switches from an outsider to an insider. Let $p(s)$ and $p'(s)$ denote the price paid by a winner when bidder 1 bids $\beta_1(s_1)$ as an outsider and bids $v_1(s)$ as an insider, respectively, whereas bidder 2 bids $\beta_2(s_2)$. Note that $p(s)$ is the seller's (ex-post) revenue in the standard setup, and $p'(s)$ is the seller's revenue in the setup in which bidder 1 is an insider.

***Corollary* 1:** $p'(s) \geq p(s), \quad \forall s \in [0, 1]^2$. That is, the seller's revenue increases relative to the standard setup as bidder 1 becomes an insider.

Proof. Note that the object is efficiently allocated whether bidder 1 bids $\beta_1(s_1)$ or $v_1(s)$. Consider first the case in which $v_1(s) > v_2(s)$, so bidder 1 wins the object. Then, $p(s) = p'(s) = \beta_2(s_2)$ because bidder 2's strategy is unchanged. Consider next the case in which $v_1(s) < v_2(s)$, so bidder 2 is a winner and pays bidder 1's bid. Thus, $v_1(s) \geq \beta_1(s_1), \ \forall$. This is obvious in the first two cases of Equation (2). In the third case, we have $\alpha \in [0, 1]$ such that $\beta_1(s_1) = v_1(s_1, \alpha) = v_2(s_1, \alpha)$. The assumption that $v_1(s_1, s_2) < v_2(s_1, s_2)$ implies that $\alpha < s_2$ because of the single-crossing property. Thus, $\beta_1(s_1) = v_1(s_1, \alpha) \leq v_1(s)$ as desired.

Q.E.D.

This result can be understood in light of the *winner's curse.* If a bidder is an insider, he estimates his rival's (unknown) signal to be only at the level that equates the two bidders' values. He does so to avoid paying more than his value when he emerges as a winner *i.e.*, getting trapped in the winner's curse. As this bidder becomes an insider and gets to know his value precisely, he no longer worries about the winner's curse and raises the bid to his true value of the object.

Let us now turn to the analysis of the first-price auction. Although no characterization of equilibrium is available, the following result shows that any equilibrium allocation must be inefficient in Case 1 and in Case 2 under mild assumption.

***Proposition* 2:** Suppose that

$$\frac{\partial v_1}{\partial s_2}, \frac{\partial v_2}{\partial s_1} > 0 \text{ (values are strictly interdependent).}$$

In the first-price auction,

(i) no efficient equilibrium exists in Case 2.

(ii) no efficient equilibrium exists in Case 1 if, for some $R>1$,

$$\frac{1}{R} < f(s) < R, \forall s \in [0,1]^2. \tag{3}$$

Proof. Let us first introduce a few notations. Define a function α: $[0, \bar{s}_2] \rightarrow [0, 1]$ such that $v_1(\alpha(s_2), s_2) = v_2(\alpha(s_2), s_2)$, where $\bar{s}_2$ is the highest s_2 at which $\alpha(s_2)$ is well-defined. It is easy to verify that $\alpha(0)=0$, $\alpha(\cdot)$ is (strictly) increasing with s_2, and $\bar{s}_2>0$. For notational convenience, $v_s := v_1(\alpha(s), s) = v_2(\alpha(s), s)$ for $s \in [0, \bar{s}_2]$, and $\bar{v} := v_1(\alpha(\bar{s}_2), \bar{s}_2)$. $F_{S_2|V_1}(\cdot|v)$ and $f_{S_2|V_1}(\cdot|v)$ denote the marginal distribution of S_2 on conditional $V_1=v$ and its density, respectively. $F_{S_1|S_2}$ and $f_{S_1|S_2}$ are similarly defined.

***Proof of Part* (i):** Suppose to the contrary that an efficient equilibrium exists. Note that the efficiency requires that bidder 2 of type $s \in [0, \bar{s}_2]$ wins if and only if $s_1 < \alpha(s)$. Therefore, bidder 2 must bid v_s. If bidder 2 bids $b > v_s$, a mass of signals $s_1 > \alpha(s)$ exist such that $b > v_1(s_1, s) > v_s$, which implies that bidder 1 loses to bidder 2 because he never bids above his value, causing an inefficiency. Likewise, if bidder 2 bids $b < v_s$, a mass of signals $s_1 > \alpha(s)$ exist such that $b < v_1(s_1, s) < v_s$, which implies that bidder 2 loses to bidder 1 because bidder 1 with such a signal will outbid bidder 2, causing an inefficiency. Thus, the efficiency requires that bidder 2 of type s wins and pays v_s if and only if $s_1 > \alpha(s)$. However, $v_2(s_1, s_2) < v_s = v_2(\alpha(s), s) = v_1(\alpha(s), s)$, which means that bidder 2 incurs a loss.

***Proof of Part* (ii):** Let us first prove the following claim:[8]

[8] This claim alone almost suffices to yield an inefficiency for the generic distribution and value functions. Suppose for instance that Equation (4) holds for a given distribution and given value functions. If either the distribution or value functions are slightly perturbed, then Equation (4) will fail.

***Claim* 1:** An efficient equilibrium exists only if

$$\frac{F_{S_2|V_1}(s \mid v_s)}{f_{S_2|V_1}(s \mid v_s)} = \frac{F_{S_1|S_2}(\alpha(s) \mid s)}{\alpha'(s) f_{S_1|S_2}(\alpha(s) \mid s)}, \forall s \in [0, \bar{s}_2]. \tag{4}$$

Proof. Let $b_1:[0, \bar{v}_1] \to \mathrm{R}_+$ and $b_2:[0, 1] \to \mathrm{R}_+$ denote the bidding strategies of bidder 1 and 2, respectively, where $\bar{v}_1 = \max_{s \in [0,1]^2} v_1(s)$. Suppose that these bidding functions lead to the efficient allocation, which means that bidder 1 wins if and only if $s_1 \leq \alpha(s_2)$. The standard argument shows that b_1 and b_2 are strictly increasing and differentiable almost everywhere in $[0, \bar{v}_1]$ and $[0, \bar{s}_2]$, respectively. Furthermore, efficiency of the resulting allocation requires

$$b_1(v_s) = b_2(s), \forall s \in [0, \bar{s}_2]. \tag{5}$$

In equilibrium, bidder 1 with $v_s \in (0, \bar{v}_1)$ chooses

$$v_s \in \arg \max_{v \in [0, \bar{v}_1]} (v_s - b_1(v)) F_{S_2|V_1}(b_2^{-1}(b_1(v)) \mid v_s).$$

The first-order condition at $v = v_s$ is given by

$$-b_1'(v_s) F_{S_2|V_1}(s \mid v_s) + (v_s - b_1(v_s)) f_{S_2|V_1}(s \mid v_s) \frac{b_1'(v_s)}{b_2'(s)} = 0,$$

as $b_2^{-1}(b_1(v_s)) = s$ from Equation (5). Thus, we have the following expression after rearrangement

$$\frac{F_{S_2|V_1}(s \mid v_s)}{f_{S_2|V_1}(s \mid v_s)} = \frac{(v_s - b_1(v_s))}{b_2'(s)}. \tag{6}$$

As for bidder 2 with $s \in [0, \bar{s}]$, because of Equation (5), he must choose

$$s \in \arg \max_{s' \in [0, \bar{s}_2]} E_{S_1|S_2}[(v_2(S_1, s) - b_2(s')) 1_{\{S_1 \leq \alpha(s')\}}]$$
$$= \int_0^{\alpha(s')} v_2(t, s) dF_{S_1|S_2}(t \mid s) - F_{S_1|S_2}(\alpha(s') \mid s) b_2(s').$$

The first-order condition at $s'=s$ after rearrangement is given by

$$\frac{F_{S_1|S_2}(\alpha(s)\mid s)}{\alpha'(s)f_{S_1|S_2}(\alpha(s)\mid s)}=\frac{(v_s-b_1(v_s))}{b_2'(s)}, \tag{7}$$

because $v(\alpha(s), s)=v_s$ and $b_2(s)=b_1(v_s)$ from Equation (5). Combining Equation (6) and Equation (7) yields Equation (4).

Q.E.D.

For the proof of Part 2, a contradiction to Equation (4) is drawn by showing that for any s, all other terms than $F_{S_1|S_2}(\alpha(s)|s)$ are bounded away from zero. First, the assumptions on value functions imply that for some finite $K>1$,

$$\frac{1}{K}<\frac{\partial v_j}{\partial s_i}(s)<K \text{ for all } i, j \text{ and } s\in[0,1]^2, \tag{8}$$

which, in turn, implies that for some finite $L>1$,

$$\frac{1}{L}<\alpha'(s)=\frac{\dfrac{\partial v_2}{\partial s_2}-\dfrac{\partial v_1}{\partial s_2}}{\dfrac{\partial v_1}{\partial s_1}-\dfrac{\partial v_2}{\partial s_1}}<L \text{ for all } s\in[0,\bar{s}_2]. \tag{9}$$

We also observe from Equation (3) that for all $(s_1, s_2)\in[0, 1]^2$ and $s\in[0, \bar{s}_2]$,

$$\frac{1}{R^2}<f_{S_1|S_2}(s_1\mid s_2),\ f_{S_2|V_1}(s\mid v_s)<R^2 \tag{10}$$

We now show that $F_{S_2|V_1}(s|v_s)$ is bounded away from zero. Fix an arbitrary $s\in[0, \bar{s}_2]$ and define $A=\{s|s_2\leq s$ and $v(s)=v_s\}$ and $B=\{s|s_2\geq s$ and $v(s)=v_s\}$. Subsequently, Equation (8) and Equation (9) imply that for some finite $M>0$, $\int_B 1<M\int_A 1$.[9] Therefore,

[9] Note that each integral measures the area of A and B. This inequality follows from the fact that the slopes of curve $\alpha(\cdot)$ and indifference curve $v(s)=v_s$ are both bounded away from zero and bounded above.

$$F_{S_2|V_1}(s\mid v_s)=\frac{\int_A f(s)}{\int_A f(s)+\int_B f(s)}=\frac{1}{1+\frac{\int_B f(s)}{\int_A f(s)}}>\frac{1}{1+\frac{R^2\int_B 1}{\int_A 1}}>\frac{1}{1+MR^2}, \tag{11}$$

where the first inequality follows from Equation (3) and the second from the fact that $\int_B 1 < M\int_A 1$. From Equation (9), Equation (10), and Equation (11), $F_{S_2|V_1}(s|v_s)$, $f_{S_2|V_1}(s|v_s)$, $f_{S_1|S_2}(\alpha(s)|s)$, and $\alpha'(s)$ are all bounded away from zero for any s. However, considering Equation (8) and Equation (9), if s is small, then for any $\varepsilon>0$,

$$F_{S_1|S_2}(\alpha(s)\mid s)=\int_0^{\alpha(s)} f_{S_1|S_2}(s_1\mid s)ds_1<R^2Ls<\varepsilon.$$

Q.E.D.

The inefficiency result is obtained rather easily for Case 2 because bidder 1, who is certain about the opponent's signal and his bid, can outbid him whenever profitable. This yields bidder 2 a loss, provided the equilibrium allocation is efficient. This argument is unavailable in Case 1 where bidder 1 knows v_1 only but not the entire signal profile, for bidder 1 is now uncertain about the opponent's bid. The proof of inefficiency for Case 1 is as follows at an intuitive level: bidder 2 with any given signal can ignore the possibility that bidder 1 has a value close to 0 and submits a very low bid, which follows from the assumption of the bounded density given in Equation (3). By contrast, bidder 1 with a value close to 0 should predict with a probability bounded away from 0 such that bidder 2 observes a signal close to 0 and submits a very low bid. Thus, he will bid much less aggressively than bidder 2 with a signal close to 0, causing an inefficiency. Note that the assumption in Equation (3) is only necessary for establishing that condition Equation (4) cannot be satisfied. Even without this assumption, the inefficiency result will hold generically because Equation (4) can only be satisfied by non-generic value distributions.

IV. Concluding Remarks

The inefficiency result of the first-price auction presented in this paper is not surprising considering that it is much harder in general for asymmetric bidders to coordinate their bids toward the efficient allocation in the first-price auction. Quantifying the magnitude of inefficiency of the first-price auction may thus be more important. This step would require characterizing the equilibrium of the first-price auction with an insider in the interdependent value setup, which is left for future research. Moreover, one can ask if the efficiency of the second-price auction will hold in the setup with more than two bidders. Choi *et al.* (2015) provides a negative answer to this question; they show that an English auction admits an efficient equilibrium. We expect that the inefficiency of the first-price auction will persist and probably be aggravated as the number of bidders increases.

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