Price Competition in a Mixed Oligopoly Market

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Several studies on mixed oligopoly indicate that the ownership pattern of firms does not affect the equilibrium price. This idea often suggests that ownership is irrelevant. In a mixed duopoly under price competition, firm ownership is irrelevant. This study reveals that ownership is irrelevant in a single publicly owned firm and in any positive number of privately owned firms. However, if two or more publicly owned firms exist, then ownership becomes relevant in a homogeneous good market with a strictly increasing convex cost schedule and a downward sloping demand curve. If firms set the price sequentially and if the lone public firm is a price leader, then social welfare is constantly greater than when the latter is a price follower. The unique price is the competitive price when the public firm moves first in the sequential game.

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I. Introduction

In recent years, numerous studies have investigated mixed oligopoly markets where public and private firms co-exist. In such a market, privately owned firms maximizing profit compete with publicly owned firms. In particular, the standard practice in the literature has been to consider the maximization of social welfare as the objective of publicly owned firms. Social welfare is often characterized as the sum of producer and consumer surpluses. Although the mixed oligopoly market is similar to oligopoly market, the essential difference lies in the objective of a few firms, namely, public firms. Evidence of mixed oligopoly can be observed in the market for products in the oil and gas, telecommunication, and iron and steel sectors.[1]

From De Fraja, and Delbono (1989), numerous studies on mixed oligopoly have assessed the quantity competition among firms, including (Delbono, and Denicolo 1993; Freshtmanm 1990; Fjell, and Heywood 2002; Fjell, and Pal 1996; Han, and Ogawa 2008; Majumdar, and Pal 1998; Pal, and White 1998; Nett 1994; Matsumura 1998). Most of the results related to quantity competition in mixed oligopoly have indicated that social welfare may increase when the objective of publicly owned firms is to maximize profit as well.

Other studies have analyzed government interventions, through quantity (output) subsidy, in the mixed oligopoly context. White (1996) concluded that ownership is irrelevant if the optimal subsidy quantity is provided before and after the privatization of public firms. If private oligopoly in a situation where public firms have been privatized is not subsidized, then social welfare falls. Poyago-Theotoky (2001) extended this result for a sequential move game where the public firm is the Stackelberg leader. Myles (2002) extended White (1996) result for a general inverse demand function and aconvex cost function. Another strand of literature on mixed oligopoly studies the government's optimal shareholding in firms. Huang, Lee, and Chen (2006) suggested that the government should adopt a strategy of mixed oligopoly when a public firm's production cost is less than or equal to that of a private firm, or

[1] For example, in the telecommunication sector in India, Mahanagar Telephone Nigam Limited and Bharat Sanchar Nigam Limited are public sector firms, whereas Bharti Airtel, TATA Telecommunications, and Reliance Communications are private sector firms operating in this country. Other similar examples are observed in the oil and gas sector, as well as the iron and steel industry.

at a threshold level. Thus, the government should privatize the public firm when this firm's production cost is considerably high.

The endogenous timing of public and private firms' actions in quantity competition has also been analyzed. Pal (1998) argued that a public firm should be a price follower and produce zero quantity. Thus, a public firm's credible threat of entry is sufficient for the enhancement of welfare. Lu (2006) extended the aforementioned result by including foreign private firms. Matsumura (2003) showed that in a duopoly market, a public firm should be the leader in equilibrium if the other firm is a foreign private firm and a follower if the other firm is a domestic firm.

Ogawa, and Kato (2006) examined the price competition[2] in a homogeneous good mixed oligopoly market, and established that ownership is irrelevant. The set of equilibrium prices is the same as that of the Bertrand competition for quadratic cost function. Under certain conditions in private firm leadership, the equilibrium price exceeds that of the simultaneous move game. Price is constantly low in the case of a public firm leadership. Dastidar, and Sinha (2011) analyzed the price competition between a public and a private firm in a homogeneous good market. The aforementioned researchers showed that ownership is irrelevant in a homogeneous good duopoly market in a general setup characterized by a downward sloping demand function and a strictly increasing convex cost function. If both firms are publicly owned, then the equilibrium set is a superset of the set of equilibria in a mixed oligopoly. In a sequential move game, if the public firm is a price leader, then the equilibrium price is the same as that of the competitive equilibrium. Moreover, if the private firm is a price leader, then the outcome is the same as that of the collusive outcome.

Our analysis closely follows Dastidar, and Sinha (2011). We have extended the duopoly model to oligopoly by retaining the general downward sloping demand function and a strictly increasing convex cost function. Moreover, we demonstrate that if a sufficient number of public firms is present out of the total number of firms, then the irrelevance of ownership may not hold. In this paper, Section I discusses the model used in this study. Section II presents the results. Section III analyzes the outcome when firms set price sequentially.

[2] Studies have investigatedprice competition in differentiated good mixed oligopoly market. However, we are not reviewing these studies because we are analyzing the homogeneous market.

II. Model

Suppose m firms ($m>2$) are present, with each one producing a homogeneous product. The firms compete among themselves in terms of price. First, we analyze the equilibrium strategy of the firms when all of them are privately owned. The privately owned firms intend to maximize their profits. *The firms have to supply the quantity demanded at a price*. The firms setting the same price will share the quantity of goods supplied equally among them.

Thereafter, we introduce a few public firmsout of the m firms in the market. These publicly owned firms aim to maximize social welfare. Suppose that $\bar{n}$, ($\bar{n}<m$) publicly owned firms are present. We analyze the effect of an increase in the number of publicly owned firms (more than $\bar{n}$) on the outcome in terms of strategies, that is, the set of Nash equilibria.

The demand function of the market is denoted as follows:

$D(p)=Q$ twice continuously differentiable.
A unique p^{max} exists, such that $D(p^{max})=0$. A unique Q^{max} also exists, such that $D(0)=Q^{max}$ and $D'(p)<0$, $\forall p\in(0, p^{max})$.

The cost function of each firm is assumed to be the same. The nature of the cost function of each firm is given as follows:

$C(x)$ is twice continuously differentiable and has the following properties, $C(0)=0$, $C'(Q)>0$, $C''(Q)>0$, and $C'(0)<p^{max}$.

If p is the lowest price set by all the privately owned firms ($n\geq 1$) in the market, then we define the profit of each privately owned firm ($n\geq 1$):

$$\pi(p) = p\frac{D(p)}{n} - C(\frac{D(p)}{n}),\ p\in[0,\ p^{max}].$$

The social welfare in the case when $n\geq 1$ firms set the lowest price p is defined asfollows:

$$W(p) = \int_{p}^{p^{max}} D(x)dx + n\pi(p)$$

The equilibrium is necessarily non-unique (Dastidar 1995) in the case of the Bertrand competition between two firms in a homogeneous product market. We show that if a few (more than one) publicly owned firms are present in a homogeneous product market serviced by a total of m firms, then the set of equilibrium prices is the same as the set of equilibrium prices in the Bertrand competition among m privately owned firms. Suppose that n^* publicly owned firms are present in the m firms market. Thereafter, we show that if n^* increases, keeping m fixed (*e.g.* private firms being nationalized), then the range of equilibrium prices shrinks. Thus, the range of equilibrium prices depends on the number of publicly owned firms.

III. Results

***Lemma* 2.1**

i) $\exists$ *unique* $\hat{p}_m \in [0, p^{max})$, such that $\hat{\pi}_m(p) = 0$.

ii) $\exists$ *unique* $\bar{p}_m \in [0, p^{max})$, such that $\bar{\pi}_m(p) = \pi(p)$.

iii) $\bar{\pi}_m(p) > \pi(p)$ if and only if $p < \bar{p}_m$.

iv) The range of the Bertrand equilibria is $p \in [\hat{p}_m, \bar{p}_m]$ when m firms are present.

Proof. See Dastidar (1995) pages 22 to 27 for the proofs.

Lemma 2.1 provides the equilibrium strategies of each firm when all of the m firms are privately owned.

***Lemma* 2.2**

$\hat{W}_m(p) > \hat{W}_{\bar{n}}(p) > W(p), \forall p \in [0, p^{max})$.

Proof. Suppose $\hat{W}_{\bar{n}}(p) - W(p) < 0, \forall p \in [0, p^{max})$.

$$\Rightarrow \int_p^{p^{max}} D(x)dx + \bar{n}\hat{\pi}(p) - \int_p^{p^{max}} D(x)dx - \pi(p) < 0.$$

$$\Rightarrow pD(p) - \bar{n}C(\frac{D(p)}{\bar{n}}) - pD(p) + C(D(p)) < 0.$$

$$\Rightarrow C(D(p)) - \bar{n}C(\frac{D(p)}{\bar{n}}) < 0.$$

By assumption, we know that the $C(\cdot)$ function is convex.

$$\Rightarrow \frac{C(D(p)) - C(\frac{D(p)}{\bar{n}})}{(\bar{n}-1)\frac{D(p)}{\bar{n}}} > \frac{c(\frac{D(p)}{\bar{n}}) - c(0)}{\frac{D(p)}{\bar{n}}}.$$

$$\Rightarrow C(D(p)) - C(\frac{D(p)}{\bar{n}}) > (\bar{n}-1)C(\frac{D(p)}{\bar{n}}).$$

$$\Rightarrow C(D(p)) - \bar{n}C(\frac{D(p)}{\bar{n}}) > 0.$$

The aforementioned inequality is a contradiction because $\bar{n}>1$.

$$\therefore \hat{W}_{\bar{n}}(p) - W(p) > 0, \forall p \in [0, p^{max}).$$

Suppose $\hat{W}_m(p) - \hat{W}_{\bar{n}}(p) < 0$.

$$\Rightarrow \int_p^{p^{max}} D(x)dx + m\hat{\pi}_m - \int_p^{p^{max}} D(x)dx - \bar{n}\hat{\pi}_{\bar{n}}(p) < 0.$$

$$\Rightarrow mp\frac{D(p)}{m} - mC(\frac{D(p)}{m}) - \bar{n}\frac{pD(p)}{\bar{n}} + \bar{n}C(\frac{D(p)}{\bar{n}}) < 0.$$

$$\Rightarrow \bar{n}C(\frac{D(p)}{\bar{n}}) - mC(\frac{D(p)}{m}) < 0.$$

We know that the $C(\cdot)$ function is convex.

$$\Rightarrow \frac{C(\frac{D(p)}{\bar{n}}) - C(0)}{\frac{D(p)}{\bar{n}}} > \frac{C(\frac{D(p)}{m}) - C(0)}{\frac{D(p)}{m}}.$$

$$\Rightarrow \bar{n}C(\frac{D(p)}{\bar{n}}) - mC(\frac{D(p)}{m}) > 0.$$

The aforementioned inequality is a contradiction because $m>\bar{n}$.

$$\therefore \hat{W}_m(p) - \hat{W}_{\bar{n}}(p) > 0, \forall p \in [0, p^{max}).$$

Thus, $\hat{W}_m(p) > \hat{W}_{\bar{n}}(p) > W(p), \forall p \in [0, p^{max})$.

From Lemma 2.2, we can see that the social welfare is an increasing function of the number of firms setting that same price.[3]

***Lemma* 2.3** $W(p)$ is maximized at $\bar{p}$, $\hat{W}_{\bar{n}}(p)$ is maximized at $p_{\bar{n}}$, and $\hat{W}_m(p)$ is maximized at p_m.

Proof. $\dfrac{dW(p)}{dp} = \dfrac{d(\int_p^{p^{max}} D(x)dx + \pi(p))}{dp}$.

$$= -D(p) + D(p) + pD'(p) - C'(D(p))D'(p).$$

$$= D'(p)(p - C'(D(p))).$$

For $p < C'(D(p)), \Rightarrow W'(p) > 0$ because $D'(p) < 0, \forall p \in (0, p^{max})$.

For $p > C'(D(p)), \Rightarrow W'(p) < 0$ because $D'(p) < 0, \forall p \in (0, p^{max})$.

$$\Rightarrow W'(p) = 0 \text{ at } p = C'(D(p)) = \bar{p}.$$

Following the same argument, $\hat{W}'_{\bar{n}}(p) = 0$, for $p = C'(\frac{D(p)}{\bar{n}}) = p_{\bar{n}}$ and $\hat{W}'_m(p) = 0$, for $p = C'(\frac{D(p)}{m}) = p_m$.

We have derived the prices at which social welfare is maximized for different numbers of firms setting the lowest price. Evidently,

$$\bar{p} = C'(D(p)) > p_{\bar{n}} = C'(\frac{D(p)}{\bar{n}}) > p_m = C'(\frac{D(p)}{m})$$

because $C'(\cdot) > 0, \forall Q$.

***Lemma* 2.4** $\exists$ *unique* $\bar{n}, \bar{n} < m$, such that

[3] We obtain this result by relaxing the assumption $C(0)=0$ to $C(0)\geq 0$ as well. However for simplicity, we opt to retain the assumption.

$$\frac{C(D(p)) - C(\frac{D(p)}{m})}{(m-1)\,\frac{D(p)}{m}} = C'(\frac{D(p)}{\bar{n}}).$$

Proof. $C(\cdot)$ is convex; thus,

$$\frac{C(D(p)) - C(\frac{D(p)}{m})}{(m-1)\,\frac{D(p)}{m}} > C'(\frac{D(p)}{m}).$$

$$\frac{C(D(p)) - C(\frac{D(p)}{m})}{(m-1)\,\frac{D(p)}{m}} < C'(D(p)).$$

Furthermore, $C''(Q) < 0, \forall Q.$

$\Rightarrow \exists \bar{n}, 1 < \bar{n} < m,$ such that

$$\frac{C(D(p)) - C(\frac{D(p)}{m})}{(m-1)\,\frac{D(p)}{m}} = C'(\frac{D(p)}{\bar{n}}),\ \bar{n}\ \text{ is unique.}$$

Suppose $\bar{n}$ is not unique; thus, $\breve{n}$ is another real number, such that

$$\frac{C(D(p)) - C(\frac{D(p)}{m})}{(m-1)\,\frac{D(p)}{m}} = C'(\frac{D(p)}{\breve{n}}).$$

$$\Rightarrow C'(\frac{D(p)}{\bar{n}}) = C'(\frac{D(p)}{\breve{n}})$$

This result is possible only when $\breve{n} = \bar{n}$ because $C''(\cdot) > 0.$

Therefore, $\breve{n}=\bar{n}$, $\bar{n}$ is unique.

Thus, $\bar{n}$ is the number of publicly owned firms. The number ($\bar{n}$) of publicly owned firms out of them total number of firms is such that

$$\frac{C(D(p)) - C(\frac{D(p)}{m})}{D(p)\,\frac{(m-1)}{m}} = C'(\frac{D(p)}{\bar{n}}).$$

This condition implies that $\bar{n} > 1$.

***Lemma* 2.5** For $p = C'(\frac{D(p)}{\bar{n}}) = p_{\bar{n}}, \hat{\pi}_m(p) = \pi(p)$.

Proof. Suppose

$$\frac{pD(p)}{m} - C(\frac{D(p)}{m}) - pD(p) + C(D(p)) = e,$$

such that $e > 0$, at $p = C'(\frac{D(p)}{\bar{n}}) = p_{\bar{n}}$.

$$\Rightarrow C(D(p)) - C(\frac{D(p)}{m}) = pD(p)\,\frac{(m-1)}{m} + e.$$

$$\Rightarrow \frac{C(D(p)) - C(\frac{D(p)}{m})}{D(p)\,\frac{(m-1)}{m}} = p + \frac{e}{D(p)\,\frac{(m-1)}{m}}.$$

We determine from Lemma 2.4 that

$$\frac{C(D(p)) - C(\frac{D(p)}{m})}{D(p)\,\frac{(m-1)}{m}} = C'(\frac{D(p)}{\bar{n}}).$$

We have assumed $p = p_{\bar{n}} = C'(\frac{D(p)}{\bar{n}})$.

$$\Rightarrow \frac{C(D(p_{\bar{n}})) - C(\frac{D(p_{\bar{n}})}{m})}{D(p_{\bar{n}})\,\frac{(m-1)}{m}} = p_{\bar{n}} + \frac{e}{D(p_{\bar{n}})\,\frac{(m-1)}{m}}.$$

This result is not possible because

$$\frac{C(D(p_{\bar{n}})) - C(\frac{D(p_{\bar{n}})}{m})}{D(p_{\bar{n}})\frac{(m-1)}{m}} = C'(\frac{D(p_{\bar{n}})}{\bar{n}}) = p_{\bar{n}}.$$

$$\Rightarrow e = 0. \Rightarrow \hat{\pi}_m(p_{\bar{n}}) - \pi(p_{\bar{n}}) = 0.$$

We know $\bar{p}_m$ is unique and $\hat{\pi}_m(\bar{p}_m) - \pi(\bar{p}_m) = 0$. From Lemma 2.5, we obtain $\bar{p}_m = p_{\bar{n}}$. Suppose p_n is such that $\hat{W}_n(p)$ is maximized. $\hat{W}_{\bar{n}}(p)$ is maximized at $p_{\bar{n}}$. For $1 \leq n < \bar{n}$,

$$\frac{D(p)}{n} > \frac{D(p)}{\bar{n}}$$

and $C'(\cdot) > 0$ are implied. Therefore, $p_n > p_{\bar{n}}$ for

$$p_n = C'(\frac{D(p)}{n})$$

and

$$p_{\bar{n}} = C'(\frac{D(p)}{\bar{n}}).$$

Similarly, for $m > n > \bar{n}$, we obtain $p_n < p_{\bar{n}}$.

We will show that $\exists k_{\bar{n}}$, $k_{\bar{n}} \in [0, p^{max})$, such that $\hat{W}_{\bar{n}}(p_{\bar{n}}) = \hat{W}_m(k_{\bar{n}})$. As $\bar{n} < m$; thus, $\hat{W}_m(p) > \hat{W}_{\bar{n}}(p)$, $\forall p \in [0, p^{max})$. At p^{max}, $\hat{W}_{\bar{n}}(p^{max}) = 0$ and $\hat{W}_m(p^{max}) = 0$. $\hat{W}_{\bar{n}}(p)$ is maximized at $p_{\bar{n}}$. $\hat{W}_m(p)$ is maximized at p_m. $p_m < p_{\bar{n}}$. $\hat{W}'_m(p) < 0$, for $p > p_m$. Therefore, a unique $p = k_{\bar{n}}$ exists, such that $\hat{W}_{\bar{n}}(p_{\bar{n}}) = \hat{W}_m(k_{\bar{n}})$. This result implies that $k_{\bar{n}} > p_{\bar{n}}$. k_n, $1 < n < m$ is a price, such that $\hat{W}_n(p_n) = \hat{W}_m(k_n)$.

***Lemma* 2.6** *$\exists$ unique n^**, with $m > n^* > \bar{n}$, such that $\hat{W}_{n^*}(p_{n^*}) = \hat{W}_m(k_{n^*})$ and $k_{n^*} = p_{\bar{n}} = \bar{p}_m$.

Proof. We have shown the existence of $\bar{p}_m$ in Lemma 2.1. From Lemma 2.5, we have obtained $p_{\bar{n}}$. We know that $\bar{p}_m = p_{\bar{n}}$. Now, fix n^*, such that $k_{n^*} = \bar{p}_m = p_{\bar{n}}$.

$$\hat{W}_m(k_{n^*}) > \hat{W}_{\bar{n}}(k_{n^*}), \forall p \in [0, p^{max}).$$

$$\hat{W}'_m(k_{n^*}) < 0, \text{ because } k_{n^*} = \bar{p}_m > p_m.$$

Therefore, n^* constantly exists, such that $\hat{W}_{n^*}(p_{n^*})=\hat{W}_m(k_{n^*})$, where $\hat{W}_{n^*}(p)$ is maximized at p_{n^*}. $\Rightarrow m>n^*>\bar{n}$.

n^* is unique because $\hat{W}'_m(p)<0$, for $p>p_m$, $\hat{W}_{n_1}(p)>\hat{W}_{n_2}$, $\forall p\in[0, p^{max})$, and $n_1>n_2$.

We obtained n^*, such that $\hat{W}_m(k_{n^*})=\hat{W}_{n^*}(p_{n^*})$. Suppose n^* is the number of publicly owned firms out of the m firms. We have increased the number of publicly owned firms from $\bar{n}$ to n^*. The number of privately owned firms is $m-n^*=l$. We will first analyze the best response of the n^* publicly owned firms.

The best response function of each publicly owned firms is defined by the function

$$t_i : [0, p^{max}]^{m-1} \mapsto [0, p^{max}], i = 1, 2 \ldots n^*.$$

$t_i(p)=p_{n^*}$, for each $i=1, \ldots, n^*$, where p is a vector, if $p_{min}>k_{n^*}$, and p_{min} is equal to the lowest price set by any one of the l private firms. If the lowest price set by the private firm is p_{min}, $p_{min}>k_{n^*}$, then the public firms will set a price p. $p=p_{n^*}$ because for $p>k_{n^*}$, $\hat{W}_{n^*}(p_{n^*})=\hat{W}_m(p)$.

$t_i(p)=\{p_{n^*}, k_{n^*}\}$, if $p_{min}=k_{n^*}$, $\forall i=1, \ldots, n^*$ because at $p=k_{n^*}$, $\hat{W}_{n^*}(p_{n^*})=\hat{W}_m(k_{n^*})$; thus, each public firm will be indifferent between setting $p=p_{n^*}$ or $p=k_{n^*}$.

$t_i(p)=p$, if $p_{min}<k_{n^*}$, $\forall i=1, \ldots, n^*$ because for $p<k_{n^*}$, $\hat{W}_{n^*}(p_{n^*})<\hat{W}_m(p)$; thus, each public firm will quote the same price p.

***Proposition* 2.1** Given m firms, out of which n^* firms are publicly owned and $m-n^*=l$ are privately owned, the set of Nash equilibria in price competition among all the firms is $[\hat{p}_m, k_{n^*}=p_{\bar{n}}=\bar{p}_m]$.

Proof. We have obtained the best response function for each of the publicly owned firms. The best response of the privately owned firms is as follows:

$$S_j(p) = p, \text{ if } p[\hat{p}_m, \bar{p}_m], \text{ for each } j = 1, \ldots l.$$

This response is the Bertrand range of m firms. We know that $\bar{p}_m = p_{\bar{n}} = k_{n^*}$.

Therefore, the set of Nash equilibria in the price competition among n^* public firms and $m - n^* = l$ private firms is the same as the set of Nash equilibria in the price competition in m privately owned firms.

***Remarks*:**

i) For $0 < n \leq n^*$, $\Rightarrow p_n \geq p_{n^*}$, where p_n maximizes $\hat{W}_n(p)$ and p_{n^*} maximizes $\hat{W}_{n^*}(p)$, the set of Nash equilibria in the price competition among n publicly owned firms and $m - n$ privately owned firms is the same as the set of Bertrand equilibria in m privately owned firms. Therefore, in the case where $m - 1$ private firms and one public firm are present, the set of equilibria is the same as the set of equilibria in m firms' Bertrand competition.

ii) For $m > n > n^*, \Rightarrow p_n < p_{n^*}$

$\Rightarrow k_n < k_{n^*}$, where p_n maximizes $\hat{W}_n(p)$, p_{n^*} maximizes $\hat{W}_{n^*}(p)$, k_n is such that $\hat{W}_{p_n}(p_n) = \hat{W}_m(k_n)$, and k_{n^*} is such that $\hat{W}_{p_{n^*}}(p_{n^*}) = \hat{W}_m(k_{n^*})$; the set of Nash equilibria in the price competition of n publicly owned firms and $m - n$ privately owned firms is a subset of the set of the Bertrand equilibria in m privately owned firms. $[\hat{p}_m, k_n] \subset [\hat{p}_m, \bar{p}_m]$. Therefore, the range shrinks.

The upper bound of the set of equilibrium prices decreases when the number of public firms among the total number of firms increases. Social welfare is higher when the number of public firms increases above a threshold level, and the firms quote the upper bound of the set of pure strategy Nash equilibrium price. Firms may charge considerably high prices, which are also equilibrium strategies, when the number of public firms is below a threshold level.

IV. Sequential Move Game

In this section, we compute the set of pure strategy Nash equilibria when firms set prices in a sequential order. We assume that one public firm and $(m-1)$ private firms are present. We consider two cases: (1) all $(m-1)$ private firms move first and the public firm is the second mover, and (2) the public firm moves first and all $(m-1)$ private firms move second.

A. Case 1

From Lemma 2.1, we know that $\bar{\pi}_m(p) \geq \pi(p)$, $\forall$, $p < \bar{p}_m$. The range of Bertrand equilibria is $p \in [\hat{p}_m, \bar{p}_m]$. We know from Lemma 2.3 that $W(p)$ is maximized at $p_0 = c'(D(p))$ and from Lemma 2.4 that

$$p_0 > \bar{p}_m = c'(\frac{D(p)}{\bar{n}}).$$

Suppose p_{min} is equal to the minimum of the price set by $(m-1)$ private firms in stage I (as the first mover). If $p_{min} \leq p_0 = c'(D(p_0))$, then the best response of the public firm is to set $p = p_{min}$ because $W'(p) > 0$, for $p < p_0$. Thus, the public firm will not undercut the price. We understand that if $p_{min} \leq p_0$, then the public firm in stage II (public firm as the second mover) will constantly set the same price, which is the minimum of the price set by private firms. If $p_{min} > p_0$, then the best response of the public firm is to set $p = p_0$ because $W(p)$ is maximized at $p = p_0$.

We know that $\bar{p}_2$ exists, such that

$$\pi_m = \frac{pD(p)}{m} - c(\frac{D(p)}{m}) \geq \pi_2 = \frac{pD(p)}{2} - c(\frac{D(p)}{2}).$$

We obtain $\bar{p}_2$ in a similar manner as in Lemma 2.1. From the strict convexity of the cost function, we also obtain $\bar{p}_2 < \bar{p}_m$ following the same argument as in Lemma 2.4. Consider a private firm j that sets price p_j in stage I and $p_{min} = Min\{p_i\}$, $\forall i \neq j$ is the minimum of the price set by all other private firms in stage I. Suppose $\bar{p}_2 < p_{min} \leq p_0$, then the best response for firm j is to set price $p_j < p_{min}$ because $\pi_m(p) < \pi_2(p)$, $\forall p > \bar{p}_2$. The public firm will always set $p = p_j$ because $p_j < p_0$. In stage I, the firms undercut the prices if any firm sets a price higher than $\bar{p}_2$. If any firm in stage I sets $p \leq \bar{p}_2$, then the best response for other firms in this stage is to set the same price. The argument is the same as in Proposition 2.7. The public firm in stage II will also set the same price because $\bar{p}_2 < p_0$. Thus, we determine that the set of pure strategy subgame perfect Nash equilibria is to set the same price $p_i \in [\hat{p}_m, \bar{p}_2]$, $\forall i$. An interesting result is that $[\hat{p}_m, \bar{p}_2] \subset [\hat{p}_m, \bar{p}_m]$.

B. Case 2

We know from Lemma 2.3 that $\hat{W}_m(p)$ is maximized at p_m. This p_m is

$$p_m = c'(\frac{D(p)}{m}) \text{ and } \bar{p}_m = \frac{c(D(p)) - c(\frac{D(p)}{m})}{d(p)(\frac{m-1}{m})}.$$

We know that $c(\cdot)$ is strictly an increasing convex function. Thus, we obtain

$$\bar{p}_m = \frac{c(D(p)) - c(\frac{D(p)}{m})}{d(p)(\frac{m-1}{m})} > p_m = c'(\frac{D(p)}{m}).$$

The public firm sets $p=p_m$ in stage I. All the $(m-1)$ private firms set this same p_m price in stage II because $\bar{\pi}_m(p) > \pi(p)$, $\forall p < \bar{p}_m$. Thus, we determine that a unique pure strategy Nash equilibrium exists, such that the public firm sets $p=p_m$ in stage I and all the $(m-1)$ private firms set the same price in stage II. A unique pure strategy subgame perfect Nash equilibrium exists in this two-stage game.

V. Conclusion

This study shows that in the presence of a single publicly owned firm operating along with any positive number of privately owned firms with price as the strategic variable, then the ownership pattern of firms is irrelevant. Hence, the set of equilibrium prices does not depend on the ownership pattern. When more than two firms are operating in a market and if a certain number of firms are publicly owned, then the set of equilibrium prices is a subset of the set of equilibrium prices in the Bertrand competition. The upper bound of the set of equilibrium prices is considerably low, that is, the set of equilibrium prices shrinks. This result implies that the ownership pattern matters when a sufficiently large number of publicly owned firms are operating alongside privately owned firms. Firms may probably chargesignificantly high prices when the number of public firms isnot sufficiently high, that is, not above a threshold level.

If the prices are set sequentially and if the public firm is the first mover (price leader), then a unique pure strategy subgame perfect Nash equilibrium exists. This unique price is the competitive equilibrium price when m firms service the market. If the public firm moves second, then we obtain a range of equilibrium prices.The upper bound of this range is less than the upper bound of the equilibrium prices in the Bertrand competition in *m* private firms. The pure strategy subgame perfect Nash equilibrium is necessarily non-unique.

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