Competition in Two-sided Platform Markets with Direct Network Effect

Jungsik Hyun

In light of recent trends in social networking services that encourage users of platforms to "share," "recommend," and "do activities" with others, this work analyzes platform competition in two-sided markets that exhibit direct (or within-) network effect in addition to conventional cross-network effect. Introduction of direct network effect to one group (buyer-side) in a two-sided market generates two counteracting effects: *demand-augmenting effect* and *demand-sensitizing effect.* The former allows platforms to raise buyer-side price, thereby increasing the sum of prices charged to buyers and sellers, whereas the latter causes platforms to lower them. I show that demand-augmenting effect dominates demand-sensitizing effect under the monopoly platform, whereas introducing competition between platforms under sufficient direct network effect relatively strengthens the demand-sensitizing effect, which lowers the price charged to buyers.

Keywords: Two-sided markets, Platform competition, Social networking services, Network effects

JEL Classification: D43, D85, L82, L86

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This work is based on my Master's thesis at Seoul National University. I am grateful to Professor In Ho Lee for his thoughtful guidance and suggestions. I also wish to thank Professor Jinwoo Kim and Professor Jihong Lee for their valuable comments. This work was supported by the Brain Korea 21 Program for Leading University & Students (BK 21 PLUS).

[**Seoul Journal of Economics** 2016, Vol. 29, No. 3]

I. Introduction

A. Motivation

The greatest revolution triggered by the widespread use of smartphones is that "sharing," "recommending," and "doing activities with friends" have become extremely easy via social networking services (SNS). This "sharing" function allows individuals to appreciate, work on, and evaluate their content with others by simply clicking a "share" button. This convenient function is rapidly transforming conventional content and industries. This trend is mostly aimed to generate direct network externalities.

A real-life transition resulting from the widespread use of smartphones is demonstrated in the way people play games. Before the mobile era, games were frequently played by only one player or with a few others. However, mobile games based on SNS platforms allow users to enjoy the games with their friends or acquaintances whose contacts are in their mobile phones. Players are no longer limited by time and space because they can play a game with anyone at any time through the SNS platforms that the game is based on. For example, Facebook, the social network giant with 1.3 billion users, provides a game platform in which users can complete missions and compare scores with their Facebook friends. Line and Kakao, which are mobile messenger apps with 600 million and 140 million users, respectively, also provide platforms for mobile games. Users can now enjoy playing games with friends on their list and even receive invitations from their friends to keep on playing. Thus, users invite more friends to play games, which significantly contributes to the rapid popularization of mobile games.

Another influence of the "sharing" function is demonstrated in the change in the consumption patterns of printed content, such as newspaper and books. Social networks, such as Facebook and Twitter, show articles that the friends of their users have read, recommend topics that users may be interested in based on the articles that their friends are reading, and display these articles on the top of their list. Amazon provides a review service to buyers. Buyers are encouraged to recommend good books and leave reviews for those books, which urge users to purchase books frequently. All these functions are intended to generate direct network effect.

The increasing use of content recommendation services based on the consumption pattern of users' friends has become notable in e-commerce. The movie review website "Watcha" distinguishes itself from other movie review websites by offering a sharing function. Users share their reviews with their Facebook friends, and this practice encourages users to share their movie experiences. Airbnb, the largest website in the world for people who rent out accommodations, provides a sharing function through Facebook or Google accounts. Users can share information about the accommodation facilities where they have stayed to help their friends or acquaintances find good accommodations. This website also provides a "review" from other users, which can generate accurate information as the number of users increases. Thus, these trends of "sharing" or "recommending" allow the identification of "good" products or contents, thereby leading to their frequent consumption. These trends are certainly driven by the widespread use of smartphones. If web-centric software platforms were "likely to produce changes that dwarf the revolution we have seen in the last quarter century (Evans, Hagiu, and Schmalensee (2006))", then this phrase definitely applies to "smartphone-centric" platforms at present.

Services that prompt users to frequently "share," "recommend," "invite," or "do activities" with friends or other users have a two-sided structure. For example, social networks, such as Facebook, Line, and Kakao, provide platforms where users can access, download, and play games with friends (or other users of that service). The games provided through these websites are originally produced by mobile game developers. That is, three parties are involved: content users (game players), platforms (Facebook, Line, and Kakao), and content developers (game developers). Amazon also functions as a platform. The company mediates between publishers and consumers. Facebook and Twitter do not publish news articles themselves. They simply provide platforms where news articles can be posted, and then encourage users to read articles that their friends (or other users) are reading. Airbnb is also a two-sided market because it offers an open platform for users who want to provide and find places for lodging.

Thus, the development of mobile technology primarily changes the structure of the two-sided market. Although conventional two-sided platform markets were characterized by the cross-network effect among different groups (or end users), incorporating direct network effect within a group (or within-network effect) is necessary. Despite the ubiquitous existence of this type of market structure, limited work has been conducted to examine its effect on the business strategies of platforms. A number of questions naturally emerge. Should platforms subsidize a group with direct network effect to attract more buyers, or should they penalize the group and extract additional surplus generated by direct network effect? This problem is not trivial because of the two-sided market structure. Any change in price charged to the group with a direct network effect will exert an indirect effect on the demand of the other group. Thus, the optimal price charged to the other group changes, which in turn affects the price charged to the former group. The answer depends on the competitive nature of a market.

Another important feature is that competition between platforms is becoming prevalent. Facebook and Twitter are not only vying to be the leading SNS in the world, but their contents and annexed services are competing as well. Mobile messenger services, such as Line and Kakao, are aggressively competing for mobile game platform services. Thus, analyzing platform competition in the presence of both direct and cross-network externalities is necessary.

To contribute to the recent two-sided platform market literature, this work seeks to answer the following questions: (1) *What is the optimal pricing strategy of monopoly/duopoly platform(s) where one side of a group enjoys direct network effect?* (2) *Compared with the case where no direct network effect exists, which side enjoys discount (or conversely, which side is penalized) by introducing such effect? Does the competitiveness of a market affect the result?* (3) *How does the magnitude of direct network effect affect the pricing strategy of platforms?* This study determines that competition among platforms may induce them to reduce the price charged to the group with direct network effect, and this trend is reinforced as the magnitude of direct network effect increases.

Accordingly, direct network effect is introduced in a rather conventional two-sided market. Each end user group is denoted as "buyers" and "sellers" or "buyer-side" and "seller-side" in some cases, and direct network effect is assumed to exist only in the buyer-side. Buyers in mobile game platforms can be regarded as game players, whereas sellers represent game developers. In SNS platforms, such as Facebook or Twitter, buyers are SNS users and sellers are content providers.

Introducing direct network effect into a two-sided market generates two counteracting effects on the pricing decision of a platform. On the one hand, it directly increases marginal utility on the buyer-side, thereby encouraging potential buyers who have previously not joined the platform to join it. This effect (called ***demand-augmenting effect***) allows the platform to increase the price charged to buyers. On the other hand, an increase in buyer-side price will dramatically reduce buyer-side demand for the platform compared with the case without direct network effect because the decrease in demand will be exacerbated by direct network effect. A reduction in demand implies a dwindling direct network effect, which indicates lower marginal utility for a given buyer-side price. This effect (called ***demand-sensitizing effect***) provides incentive for a platform to lower buyer-side price because reducing price will attract more buyers, which in turn will enhance direct network effect. Increased marginal utility, which Is generated by direct network effect, will attract more buyers. This study shows that competition among platforms relatively amplifies demand-sensitizing effect because competition restricts demand-augmenting effect by causing competing platforms to split total demand, whereas demand for one platform will be more sensitive to price increase because of the presence of its competitor. Demand-augmenting effect dominates under a monopoly platform, whereas demand-sensitizing effect prevails under a duopoly framework. If demand-sensitizing effect dominates, then each platform discounts buyer-side price and raises seller-side price. In this case, the sum of prices that is charged to buyers and sellers decreases. This scenario is consistent with the pricing convention of platforms under a competitive environment. These real-world platforms typically charge low fees (or even allow free usage of platform services) to customers and charge high fees to content providers. Furthermore, this scenario has particular implications for the antitrust policy. Although buyer-side prices charged by competing platforms appear "too" low, these prices may simply reflect a strong direct network effect and not anticompetitive predatory pricing or dumping.

The remaining parts of this paper are organized as follows. The pricing decisions of a monopoly platform are analyzed in Section II. In Section III, the competition between two platforms is modeled à la Hotelling. Comparative statistics are provided in Section IV. I conclude in Section V.

B. Related Literature

The current work is closely related to several strands of existing research. The first strand is related to general price theory on two-sided platform markets (mostly focusing on monopoly platform). The second strand focuses on the competition issue in two-sided markets. Third, this work is related to literature analyzing direct network externalities (Katz, and Shapiro 1985; Liebowitz, and Margolis 1994). Finally, this paper is in line with literature on oligopoly market focusing on interaction among competing firms (Bulow *et al.*, 1985; Ryu, and Kim 2011).

The pioneering works of Rochet, and Tirole (2003, 2006) and Armstrong

(2006) describe price theory on two-sided platforms in the absence of direct network effect. In Rochet, and Tirole (2003), total price, which is defined as the sum of prices imposed to each side, is given by Lerner's formula, and the price structure (*i.e.*, prices imposed to each side) is provided by the ratio of the demand elasticity of each side. Although the prices considered by Rochet, and Tirole (2003) are a per-transaction-based "usage fee," they incorporate interaction-independent fixed fees called "membership charges" into the model.

Weyl (2010) develops a general theory on pricing decision for a multi-sided platform market. He shows how a monopoly platform sets prices in a multi-sided market, where agents have a general form of utility that subsumes those considered in Rochet, and Tirole (2003, 2006) and Armstrong (2006). The general utility considered in his model subsumes the possible existence of both direct and cross-network effects. He transforms the problems of monopoly platforms from price selection to desired allocations by introducing insulating tariffs to avoid coordination failure. Weyl demonstrates that even in this general framework, the profit-maximizing allocations and prices of monopolists cause classical market power distortion; however, distortion is generated by only internalizing network externalities to marginal users (Spence 1975). However, he focuses on providing a general theory and not specifically on direct network effect. Thus, Weyl does not explicitly investigate how the introduction of direct network effect changes the pricing decisions of platforms compared with the case where no effect occurs. He also does not consider competition among multiple platforms, which is more consistent with real-world platform markets.

Modeling competition between two platforms is a difficult task given the nature of two-sidedness. Rochet, and Tirole (2003) note that many merchants accept both Amex and Visa cards, and some buyers use both cards. This possibility of *multi-homing* either sides of end users complicates the illustration of competition among multiple platforms. Rochet, and Tirole (2003) address this issue by constructing a formal model that captures the nature of competing platforms. Suppose that one platform offers a lower price to sellers than its rival. Each seller must choose whether to join the cheaper platform or both platforms. A trade-off occurs because if the seller joins both platforms, he/she can transact with a larger subset of buyers. However, less buyers use the platform that offers a cheaper price to the seller compared with the case where the seller only joins the cheaper platform. Each competing platform can encourage sellers to stop multi-homing and join only its platform, which is called "steering," by undercutting the rival platform. In the current study, homogeneous products are assumed to avoid multi-homing issue because this work aims to clarify the direct effect on the pricing decision of platforms in the presence of direct network effect. Incorporating multi-homing issue is a potential future research topic.

Caillaud, and Jullien (2003) analyze the intermediation market by focusing on cross-network externalities, non-exclusivity of services, and price discrimination, which are relevant for informational intermediation via the Internet. They formulate the problem as an imperfect competition between two matchmakers in the presence of cross-network externalities where matched end users bargain to determine how the total net trade surplus is split. They show that an equilibrium with efficient market structure exists, and that the efficient structure may involve a monopolistic intermediary or duopolistic intermediaries with non-exclusive technologies and low costs. They also demonstrate the possible existence of an inefficient equilibrium that involves multi-homing on one side and single-homing on the other side.

Although the model of Rochet, and Tirole (2003) has direct implications on the payment card industry where fees are charged on a per-transaction basis, Armstrong (2006) models the competition among platforms that charge lump-sum fees. His models are applicable to markets such as shopping malls or newspapers. Armstrong provides three models for two-sided markets: (1) a monopoly platform model, (2) a model for competing platforms where agents join a single platform, and (3) a model for competing platforms where one side of agents join all platforms (called "competitive bottlenecks"). He shows that each platform in the competitive bottleneck model where sellers multi-home only considers the joint surplus of the platform and its buyers and disregards the interests of sellers. Thus, each platform encourages few sellers to join unlike that in the social optimum.

The research question of Parker, and van Alstyne (2005) may be the closest to that in the current work. They analyze two-sidedness, which focuses on information products, and theoretically demonstrate the condition where a free-goods market (*e.g.*, streaming media companies provide consumers with free software players but charge developers to create content) may exist. Parker and van Alstyne determine which side of a two-sided market obtains a discount. They show that if the increment to profit for one complementary good exceeds the lost in profit for the other good, then a discount or subsidy becomes profit maximizing. Thus, free-goods markets can exist whenever the profit-maximizing price of zero or less generates cross-network externality benefits that are greater than intramarket losses. However, their work focuses on cross-network effect and disregards direct network effect within a group, which is prevalent in the information product market.

White, and Weyl (2012) propose a novel solution concept of *residual insulated equilibrium* for platform competition. This new solution concept resolves the "indeterminancy problem" that frequently emerges in the analysis of competition among platforms. They generalize the model to accommodate direct network effect. Thus, the work is clearly related to the current study. However, White and Weyl focus on proposing a new solution concept and resolving equilibrium indeterminancy and have not thoroughly investigated the effect of introducing direct network effect. By contrast, the current work compares platform competition with and without direct network effect, thereby focusing on equilibrium prices between the two cases. The current work adopts the framework of Rochet, and Tirole (2003) because it compares the prices between the two cases and provides comparative statistics.

II. Monopoly Platform

A. Basic Framework

Platforms that utilize smart technology, particularly platforms based on SNS, mostly charge fees on a per-transaction basis and benefit from usage.[1] The present study does not consider membership charge, which is considered in Rochet, and Tirole (2006) nor membership benefit, which is considered in Weyl (2010). A model is developed based on the work of Rochet, and Tirole (2003) because its structure can subsume the substantial realistic features of mobile content platforms. Unlike that of Rochet, and Tirole (2003), this work considers cross-network effect generated between buyers and sellers as well as direct network effect among buyers. A monopoly platform that mediates transactions between pairs of end users with buyers (superscript B) and sellers (superscript S) is provided as an example. Let the platform's marginal

[1] For example, platforms for mobile games charge fees on a per-transaction basis. If a game user buys an "item" in the game, a fraction of the item price is transferred to the platform. "Emoticons" or "stickers" can be purchased by users of messenger services. Fees are charged per-transaction (*i.e.*, whenever a messenger user buys a package of "emoticons" designed by a designer, fees are charged by the intermediating platform). Thus, platforms benefit from usage.

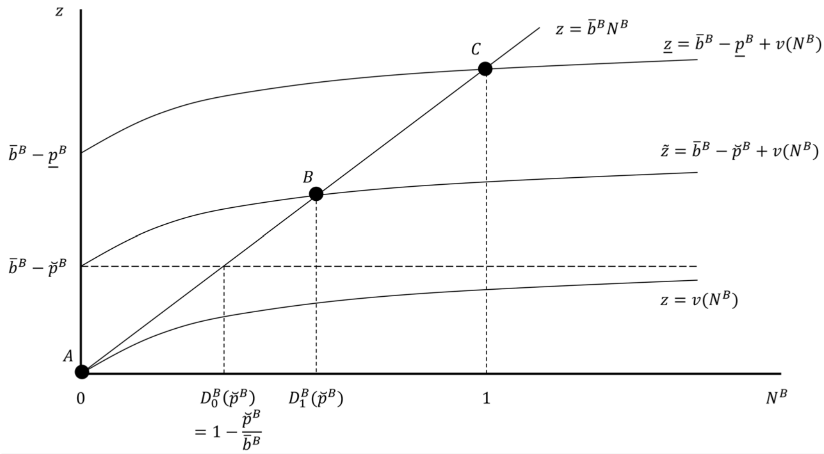

FIGURE 1
BUYER-SIDE DEMAND FOR EACH p^B

cost of transaction be given by $c > 0$. In the absence of fixed usage costs and fixed fees, the demand of the buyer-side (seller-side) depends on price p^B (p^S) imposed by the monopoly platform.

The monopoly platform chooses prices to maximize its profit. The pricing decision of the platform clearly depends on the demands of the buyer-side and seller-side for the platform (which will be formally defined in a later section), and the demand for the platform of each group is determined by thegross surplus of a group. Suppose that the gross per transaction surplus of a buyer is given by

$$\tilde{b}^B \equiv b^B + v(D_1^B(p^B)), \tag{1}$$

where b^B is a random variable that is uniformly distributed at an interval of $[0, \bar{b}^B]$, $D_1^B(p^B)$ denotes the buyer-side demand, and $v(\cdot)$ is a direct network externality function adapted from Katz, and Shapiro (1985). Assume that $v(0) = 0$, $v'(z) > 0$, and $v''(z) \leq 0$ for all $z \in [0, 1]$. Assume also that $\bar{b}^B > v'(0)$. $(v'(D_1^B))/\bar{b}^B$ is the marginal benefit from the unit increase of demand. The assumption guarantees that the marginal benefit from the unit increase of demand is always less than one. This assumption effectively defines and stabilizes the demand function by bounding the size of direct network externalities.

Figure 1 illustrates how buyer-side demand $D_1^B(p^B)$ is defined and compares it with hypothetical demand D_0^B without direct network effect.

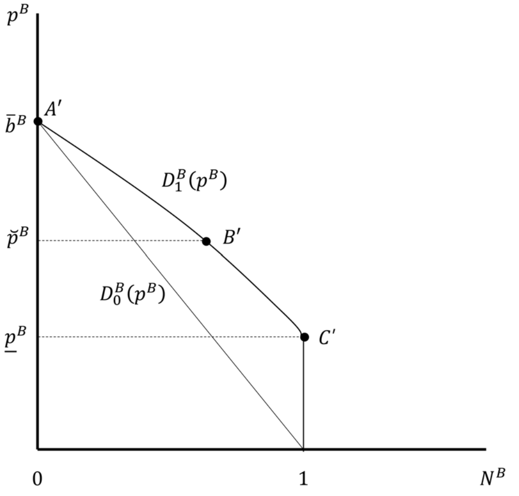

FIGURE 2
BUYER-SIDE DEMAND

The latter is defined as $D_0^B(p^B) \equiv Pr[b^B \geq p^B] = 1-(p^B/\bar{b}^B)$. If $p^B \geq \bar{b}^B$, then $D_1^B(p^B) \equiv 0$. This condition corresponds to point A in Figure 1. Let $\underline{p}^B$ be the highest price that achieves $D_1^B(\underline{p}^B)=1$. If $p^B \leq \underline{p}^B$, then $D_1^B(p^B) \equiv 1$ is defined as the upper bound of the number of buyers, which is one. This value corresponds to point C. If $p^B \in [\underline{p}^B, \bar{b}^B]$, then $D_1^B(p^B) \equiv N^B \in (0, 1]$ is uniquely determined as a fixed point of $N^B = Pr[b^B + v(N^B) \geq p^B](=1-(p^B - v(N^B))/\bar{b}^B)$ or equivalently

$$\bar{b}^B N^B = \bar{b}^B - p^B + v(N^B). \tag{2}$$

Figure 2 shows the corresponding demand curve where A', B', and C' correspond to A, B, and C in Figure 1, respectively.

The monopolist platform will not choose $p^B < \underline{p}^B$. Thus, $p^B \geq \underline{p}^B$ is assumed. The buyer-side demand exhibits the following properties:

***Remark* 1.**

(i) *Downward sloping:*

$$\frac{dD_1^B}{dp^B} = \frac{1}{\bar{b}^B - v'(D_1^B)} < 0 \textit{ for all } p^B \leq \bar{b}^B; \textit{ otherwise, } \frac{\partial D_1^B}{\partial p^B} = 0.$$

(ii) *Concavity:*

$$\frac{d^2 D_1^B}{dp^{B2}} = -\frac{v''(D_1^B)\cdot\dfrac{dD_1^B}{dp^B}}{\{\bar{b}^B - v'(D_1^B)\}^2} \leq 0 \text{ for all } p^B \leq \bar{b}^B\text{; otherwise, } \frac{d^2 D_1^B}{dp^{B2}} = 0.$$ [2]

This condition indicates the concavity of $\log D_1^B$.

Direct network effect among sellers is not considered. Thus, the gross per transaction surplus is given by b^S, which is assumed to be uniformly distributed on $[0, \bar{b}^S]$ and independent from b^B. If $p^S < \bar{b}^S$, then the demand function of the seller is defined as

$$D^S(p^S) \equiv Pr[b^S \geq p^S] = 1 - \frac{p^S}{\bar{b}^S} \in (0, 1]. \tag{3}$$

If $p^S \geq \bar{b}^S$, then $D^S(p^S) \equiv 0$.

This demand specification for each group, which is based on the work of Rochet, and Tirole (2003), has several advantages. In addition to its simplicity, this approach makes the decision of each side independent from the demand level of the other side. This condition does not indicate that cross-network effect does not exist between two groups. The net per transaction surpluses on each side are defined as $V^B(p^B) \equiv \int_{p^B}^{\bar{b}^B} D_1^B(t)dt$ and $V^S(p^S) \equiv \int_{p^S}^{\bar{b}^S} D^S(t)dt$. By assuming that b^B and b^S are independent, which is consistent with with the work of Rochet, and Tirole (2003), the average net surplus on each side is given by

$$W^B(p^B, p^S) \equiv V^B(p^B)\cdot D^S(p^S), \tag{4}$$

$$W^S(p^S, p^B) \equiv V^S(p^S)\cdot D_1^B(p^B). \tag{5}$$

As the demand on one side (*e.g.*, seller-side) increases, the other side (*e.g.*, buyer-side) will have more opportunities to transact, which increases the average net surplus of the latter side (buyer-side). This condition captures cross-network effect among end users. However, in the demand specification from Rochet, and Tirole (2003), the decision of each end user becomes independent from the level of the other side's demand, whereas the monopoly platform considers cross-network effect in deciding

[2] If $v''(z) < 0$ $\forall z \in [0, 1]$, then the inequality becomes strict.

price. The current work focuses on how network externalities (both direct and cross) affect the pricing decision of a platform. Thus, this demand specification appears sufficient to capture the concepts to be analyzed in this study.

B. Pricing Decision

By assuming that b^B and b^S are independent, the probability that the monopoly platform (charging p^B, p^S to each side) successfully mediates the transaction between the two groups is given by $D_1^B(p^B)\times D^S(p^S)\in[0, 1]$. Per-transaction profit is given by (p^B+p^S-c). Thus, the monopoly platform chooses prices to maximize the total profit as follows:

$$\pi=(p^B+p^S-c)D_1^B(p^B)D^S(p^S).$$

D_1^B and D^S are log concave.[3] Thus, this maximization problem is characterized by the first order conditions (FOCs), *i.e.*,

$$\frac{\partial(\log\pi)}{\partial p^B}=\frac{1}{p^B+p^S-c}+\frac{1}{D^B(p^B)}\frac{dD^B(p^B)}{(dp^B)}=0,$$

$$\frac{\partial(\log\pi)}{\partial p^S}=\frac{1}{(p^B+p^S-c)}+\frac{1}{D^S(p^S)}\frac{dD^S(p^S)}{(dp^S)}=0.$$

Marginal cost c is sufficiently small. Thus, the monopoly platform never chooses $p^B\geq\bar{b}^B$ or $\underline{p}^S\geq\bar{b}^S$. The FOCs are written as

$$p^B+p^S-c=D_1^B(p^B)[\bar{b}^B-v'(D_1^B(p^B))], \tag{6}$$

$$p^B+p^S-c=\bar{b}^S-p^S. \tag{7}$$

Prices p^B and p^S, which solve Equations (6) and (7), characterize the optimal pricing decision of the monopoly platform, which are denoted as $\hat{p}^B$ and $\hat{p}^S$.

By defining the elasticity of buyer-side demand D_1^B as

$$\varepsilon_1^B(p^B)\equiv-\frac{p^B}{D_1^B}\frac{dD_1^B}{dp^B}=\frac{p^B}{D_1^B[\bar{b}^B-v'(D_1^B)]}$$

[3] See Remark 1 (ii).

and that of sellers as

$$\varepsilon^S(p^S) \equiv -\frac{p^S}{D^S}\frac{dD^S}{dp^S} = \frac{p^S}{\bar{b}^S - p^S},$$

we obtain a generalized result of Proposition 1 from Rochet, and Tirole (2003) under direct network effect among buyers. Equations (6) and (7) can be written as

$$p^B + p^S - c = p^B / \varepsilon_1^B(p^B),$$
$$p^B + p^S - c = p^B / \varepsilon^S(p^S).$$

For notational simplicity, denote $\hat{\varepsilon}_1^B \equiv \varepsilon_1^B(\hat{b}^B)$ and $\hat{\varepsilon}^S \equiv \varepsilon^S(\hat{p}^S)$. Assume that the total volume elasticity in the equilibrium $\hat{\varepsilon}_1 \equiv \hat{\varepsilon}_1^B + \hat{\varepsilon}^S$ exceeds one. Then, the following proposition is obtained.

***Proposition* 1.**

(i) The total price of the monopoly platform $\hat{p} \equiv \hat{p}^B + \hat{p}^S$ is given by the standard Lerner formula for the total volume elasticity as follows:

$$\frac{(\hat{p} - c)}{\hat{p}} = \frac{1}{\hat{\varepsilon}_1}. \tag{8}$$

(ii) The price structure is given by the ratio of elasticities as follows:

$$\frac{\hat{p}^B}{\hat{\varepsilon}_1^B} = \frac{\hat{p}^S}{\hat{\varepsilon}^S}. \tag{9}$$

The closed form equilibrium prices are provided in Appendix 1. The characterization of the equilibrium prices is described in detail in the next subsection.

C. With Direct Network Effect versus Without Direct Network Effect

This study focuses on the effect of introducing direct network externalities on the optimal behavior of platforms. Following the convention of Parker, and van Alstyne (2005), the optimal prices under both network effects are compared only with those with cross-network effect. The two countering pressures on buyer-side price caused by direct network effect will be demonstrated by comparing the equilibrium prices of the two cases. This pressure on buyer-side price transmits to seller-side price in the opposite direction because of cross-network effect.

The equilibrium prices of the benchmark case (*i.e.*, without direct network effect) are derived by assuming $v(z)\equiv 0,\ \forall z\in[0, 1]$.

$$\tilde{p}^B=\frac{1}{3}[2\bar{b}^B-\bar{b}^S+c], \qquad \tilde{p}^S=\frac{1}{3}[2\bar{b}^S-\bar{b}^B+c], \qquad \tilde{p}=\frac{1}{3}[\bar{b}^B+\bar{b}^S+c]. \quad (10)$$

The elasticity of the buyer-side demand in the benchmark case is defined as

$$\varepsilon_0^B(p^B)\equiv -\frac{p^B}{D_0^B}\frac{dD_0^B}{dp^B}=\frac{p^B}{\bar{b}^B-p^B};$$

and $\varepsilon_0^B(\tilde{p}^B)$, $\varepsilon_1^B(\tilde{p}^B)$, $\varepsilon^S(\tilde{p}^S)$ are denoted as $\tilde{\varepsilon}_0^B$, $\tilde{\varepsilon}_1^B$, $\tilde{\varepsilon}^S$, respectively.

In the case where both direct and cross-network effects occur, equilibrium prices $\hat{p}^B$ and $\hat{p}^S$ are characterized by Proposition 1. The following theorem indicates who enjoys a discount and who is penalized by introducing direct network effect and compares these conditions with the case where this effect does not exist in a monopolistic two-sided platform market. $\tilde{\varepsilon}_0^B$, $\tilde{\varepsilon}_1^B$, and $\tilde{\varepsilon}^S$ are the elasticities evaluated at benchmark equilibrium prices $\tilde{p}^B$ and $\tilde{p}^S$, whereas $\hat{\varepsilon}_1^B$ and $\hat{\varepsilon}^S$ are those evaluated at the equilibrium prices when both direct and cross-network effects $\hat{p}^B$ and $\hat{p}^S$ occur. Define $p\equiv p^B+p^S$.

***Theorem* 1.**

Solutions $\hat{p}^B$ and $\hat{p}^S$ can be used to solve Equations (8) and (9). Moreover, $\hat{p}^B\geq\tilde{p}^B$, $\hat{p}^S\leq\tilde{p}^S$, and $\hat{p}\geq\tilde{p}$ hold, where equalities are satisfied if and only if

$$\frac{v(D_1^B(\tilde{p}^B))}{D_1^B(\tilde{p}^B)}=v'(D_1^B(\tilde{p}^B)).$$

Proof. The existence and uniqueness of the solution are demonstrated in an alternative proof provided in Appendix 2. This proof shares a symmetric logic to that of Theorem 2 (duopoly case). Thus, the logic of the proof of Theorem 2, which may be complicated because of the extensive calculation, is clarified.

A simpler version, which exploits the closed form equilibrium prices

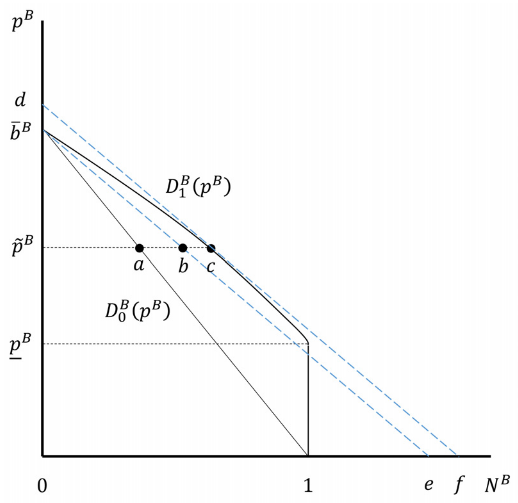

FIGURE 3
BUYER-SIDE DEMAND

provided in Appendix 1, is provided given the existence and the uniqueness of the solution. Assume $\hat{p}^B < \tilde{p}^B$. Thus,

$$\begin{aligned}\hat{p}^B &= \frac{1}{3}\{2\bar{b}^B - \bar{b}^S + c\} + \frac{2}{3}\{v(\hat{D}_1^B) - \hat{D}_1^B \cdot v'(\hat{D}_1^B)\}\\ &= \tilde{p}^B + \frac{2}{3}\{v(\hat{D}_1^B) - \hat{D}_1^B \cdot v'(\hat{D}_1^B)\}\\ &< \tilde{p}^B.\end{aligned}$$

However, $(v(\hat{D}_1^B))/(\hat{D}_1^B) \geq v'(\hat{D}_1^B)$ holds based on Jensen's inequality, which is contradicting. Thus, $\hat{p}^B \geq \tilde{p}^B$ holds.

$\hat{p}^S \leq \tilde{p}^S$ and $\hat{p} \geq \tilde{p}$ can be similarly shown. □

In the alternative proof provided in Appendix 2, $\tilde{\varepsilon}_1^B < \tilde{\varepsilon}_0^B$ always holds in a monopoly platform case. The reason why buyer-side elasticity decreases for given price $\tilde{p}^B$ is graphically illustrated in Figure 3. For given price $\tilde{p}^B$, the elasticity of D_0^B at point a is $\tilde{\varepsilon}_0^B$. The elasticity of D_1^B at point c is $\tilde{\varepsilon}_1^B$. A linear demand curve connects ($\bar{b}^B$ *be*). The elasticity of ($\bar{b}^B$ be) at b is equal to that of D_0^B evaluated at a. Thus, the following holds:

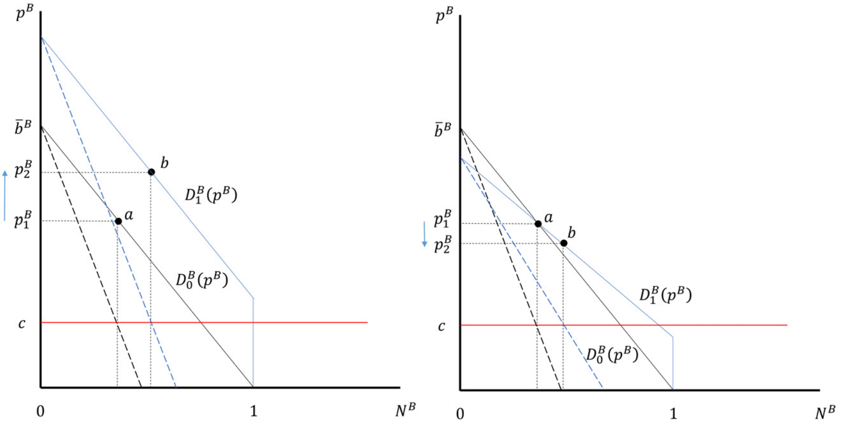

FIGURE 4
DEMAND-AUGMENTING EFFECT *VS.* DEMAND-SENSITIZING EFFECT

$$\tilde{\varepsilon}_0^B = \varepsilon^B_{(\bar{b}^B\ be)} = \frac{\overline{(\tilde{p}^B, 0)}}{\overline{(\bar{b}^B, \tilde{p}^B)}} > \frac{\overline{(\tilde{p}^B, 0)}}{\overline{(d, \tilde{p}^B)}} = \tilde{\varepsilon}_1^B,$$

where $\overline{(m, n)}$ denotes the length of the line that connects points m and n. Parker, and van Alstyne (2005) also point out that network effect makes demand more inelastic as the "size" of the demand increases, although the increase in size stemmed from cross-network effect in their context.

Note that $\tilde{\varepsilon}_1^B < \tilde{\varepsilon}_0^B$ can be rewritten as

$$\frac{D_1^B(\tilde{p}^B)}{D_0^B(\tilde{p}^B)} > \frac{dD_1^B(\tilde{p}^B)/dp^B}{dD_0^B(\tilde{p}^B)/dp^B}.$$

In the alternative proof in Appendix 2, $D_1^B(\tilde{p}^B) > D_0^B(\tilde{p}^B)$ and $\bar{b}^B - v'(D_1^B(\tilde{p}^B)) < \bar{b}^B$ always hold, and the second inequality can be written as

$$\left(-\frac{dD_1^B(\tilde{p}^B)}{dp^B}\right) > \left(-\frac{dD_0^1(\tilde{p}^B)}{dp^B}\right).$$

Thus, direct network effect introduces two effects. For a given equilibrium price without direct network effect ($\tilde{p}^B$), introducing such effect (i) directly increases buyer-side demand and (ii) raises the absolute value of the first derivative of demand with respect to buyer-side price. The latter effect implies that under direct network effect, buyer-side demand decreases more sharply as price rises. The former is called ***demand-augmenting effect***, and the latter is called ***demand-sensitizing effect***. Thus, $\tilde{\varepsilon}_1^B < \tilde{\varepsilon}_0^B$ indicates that in a monopoly case, the demand-augmenting effect dominates the demand-sensitizing effect. Only in the case

$$\frac{v(D_1^B(\tilde{p}^B))}{D_1^B(\tilde{p}^B)} = v'(D_1^B(\tilde{p}^B))$$

do the two effects balance out. These two counteracting effects on buyer-side are decomposed in Figure 4. The left panel shows how demand-augmenting effect causes platforms to increase buyer-side price. The platform has an incentive to increase p^B to extract the marginal utility of buyers generated by direct network effect. The right panel indicates how demand-sensitizing effect generates downward pressure on p^B. By reducing buyer-side price, the platform can increase buyer-side demand at a larger magnitude compared with the case without direct network effect, which is profitable.

The principle behind Theorem 1 is straightforward. $\tilde{\varepsilon}_1^B \leq \tilde{\varepsilon}_0^B$ indicates that for given equilibrium prices $\tilde{p}^B$ and $\tilde{p}^S$ without direct network effect, the introduction of direct network effect causes the elasticity of buyer-side demand to become "too" inelastic. This result demonstrates that equilibrium prices without direct network effect cannot be supported as equilibrium prices if direct network effect is introduced. Buyer-side demand becomes too inelastic because demand-augmenting effect dominates demand-sensitizing effect under the monopoly platform. Section III explains that this result this is closely related to the competitive nature of the platform market. Demand loss when a platform increases buyer-side price is limited because of monopolistic power, which results in moderate demand-sensitizing effect. Section III explains that introducing competition among platforms can reverse the relative magnitude of demand-augmenting and demand-sensitizing effects. Hence, the optimal buyer-side price under direct network effect is higher than the optimal price without direct network effect.

Meanwhile, introducing direct network effect on the buyer-side transmits to the seller-side because of cross-network effect. Under $\tilde{p}^B$ and $\tilde{p}^S$, the elasticity of seller-side demand becomes relatively more elastic than that of buyer-side demand. Hence, equilibrium prices without direct network effect can no longer be supported as the optimal price under direct network effect. The platform acquires an incentive to decrease p^S, thereby inducing a substantial increase in seller-side demand. In this manner, the platform balances the demands of the the end users to achieve Equation (9) because of the conventional cross-network effect in two-sided markets.

III. Duopoly Platforms

A. Model

The case of a duopoly, where two platforms, namely, $i=1, 2$, compete for the market, is considered in this section. The products of sellers are identical but different in location. Consequently, multi-homing does not occur unlike in the other studies mentioned earlier.

The analysis is based on a variant of Hotelling's model, where the preferences for platforms of a buyer (seller) are represented by his/her location $x(y)$ on a line. Buyer-side price imposed by platform i is denoted as p_i^B and seller-side price as p_i^S. Similar to the linear demand specification considered in Rochet, and Tirole (2003), buyers are assumed to be uniformly distributed at a close interval of $[-(\Delta/2)-\delta, (\Delta/2)+\delta]$. Furthermore, sellers are also assumed to be uniformly distributed at $[-(\Delta/2)-\delta, (\Delta/2)+\delta]$. Platforms 1 and 2 are symmetrically located at a distance $\Delta/2$ from the origin of the line. That is, platform 1 is located at $-\Delta/2$ and platform 2 at $\Delta/2$. In addition, buyers and sellers are assumed to have access to outside options, which are represented by two other symmetric platforms located at $-(\Delta/2)-\delta$ and $\Delta/2+\delta$, and denoted as platforms 0_1 and 0_2, respectively. Platform 0_i provides a net surplus b_0^B to buyers and b_0^S to sellers who join it, but does not generate direct network effect. Markets are assumed to be covered in the sense that all buyers and sellers should join at least one of the four platforms and should join only one of these platforms.

Introducing an outside option has the following advantage. If the standard Hotelling's model, where platforms 1 and 2 are located at the end points, is used, then p_i^B and p_i^S cannot be pinned down. Only the sum of prices p_i^B and p_i^S is determined. This indeterminancy mainly arises from the fact that when full coverage and symmetry of the network externality function is assumed, the demand of each platform will be $1/2$ regardless of whether direct network effect is present. This indeterminancy remains even if we introduce direct network effect. This problem can be resolve by providing outside options to each agent.

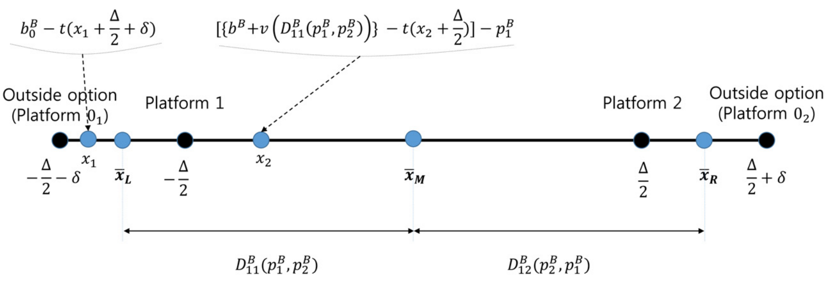

FIGURE 5
DUOPOLY PLATFORMS: BUYER-SIDE

Given p_1^B and p_2^B, denote buyer-side demand for platform i ($i=1, 2$) as $D_{1i}^B(p_i^B, p_j^B)$, where $i \neq j$. Let the gross-per-transaction surplus of a buyer joining platform i be

$$\tilde{b}_i^B \equiv b^B + v(D_{1i}^B(p_i^B, p_j^B)), \quad (i \neq j), \tag{11}$$

where $v(\cdot)$ is a common externality function that satisfies $v(0)=0$, $v'(z)>0$, and $v''(z) \leq 0$ for all $z \in [0, 1]$, and b^B is fixed across buyers. For simplicity, b^B is assumed to be independent from the platform choice of buyers.

Thus, a buyer located at x and joining platform i ($i=1, 2$) has utility

$$\{b^B + v(D_{1i}^B(p_i^B, p_j^B))\} - p_i^B - t|x - x_i|, \tag{12}$$

where t denotes a transportation cost,[4] and $x_1 \equiv -(\Delta/2)$, $x_2 \equiv \Delta/2$.

Meanwhile, if a buyer chooses the outside option platform 0_i ($i=1, 2$), then his/her net surplus will be b_0^B, but he/she will be unable to enjoy additional surplus generated by direct network effect. Thus, if a buyer located at x chooses platform 0_i, then his/her utility will be

$$b_0^B - t|x - x_{0_i}|, \tag{13}$$

where $x_{0_1} = -(\Delta/2) - \delta$ and $x_{0_2} = (\Delta/2) + \delta$. Buyers located on $[-(\Delta/2) - \delta, 0)$ will never choose platform 0_2, and those on $(0, (\Delta/2) + \delta]$ will never choose platform 0_1. b^B and b_0^B are assumed to be sufficiently large, and

[4] The term $t|x - x_i|$ can be interpreted as a measure of the dissatisfaction of buyer x with platform i.

thus, buyer-side market is fully covered.[5]

The upper left of Figure 5 depicts the net surplus of a buyer located at x_1 who chooses platform 0_1, whereas the middle depicts the net surplus of a buyer located at x_2 who chooses platform 1.

Given p_1^B and p_2^B, let $\bar{x}_L$, $\bar{x}_M$, and $\bar{x}_R$ be the locations of a buyer who is indifferent between platforms 0_1 and 1, 1 and 2, and 2 and 0_2, respectively. Then, as illustrated in Figure 5, the buyer-side demand for platform 1 is determined by $D_{11}^B\ (p_1^B, p_2^B) = (1/(2\delta+\Delta))(\bar{x}_M - \bar{x}_L)$, and that for platform 2 by $D_{12}^B = (1/(2\delta+\Delta))(\bar{x}_L - \bar{x}_M)$. That is, buyer-side demand for platform $i\ (i=1, 2)$ is implicitly determined by the following equations:

$$D_{11}^B = \frac{1}{(2\delta+\Delta)}\ \frac{1}{2t}\,[2v(D_{11}^B) - v(D_{12}^B)] + \frac{1}{(2\delta+\Delta)}\ \frac{1}{2t}\,[(p_2^B - 2p_1^B) + (b^B - b_0^B) + t(\delta+\Delta)], \tag{14}$$

$$D_{12}^B = \frac{1}{(2\delta+\Delta)}\ \frac{1}{2t}\,[2v(D_{12}^B) - v(D_{11}^B)] + \frac{1}{(2\delta+\Delta)}\ \frac{1}{2t}\,[(p_1^B - 2p_2^B) + (b^B - b_0^B) + t(\delta+\Delta)]. \tag{15}$$

For comparison, consider the case with no direct network effect. In this case, the gross-per-transaction surplus of a buyer joining either platforms 1 or platform 2 is b^B. Moreover, $v(z) \equiv 0$ for all $z \in [0, 1]$ in the case. Thus, buyer-side demand for platform $i\ (i=1, 2)$ is

$$D_{0i}^B\ (p_i^B, p_j^B) = \frac{1}{(2\delta+\Delta)}\ \frac{1}{2t}\,[(p_j^B - 2p_i^B) + (b^B - b_0^B) + t(\delta+\Delta)]. \tag{16}$$

A structure that corresponds to the assumption $\bar{b}^B > v'(0)$ in Section II is imposed. If a unit increase in demand generates "too much" marginal benefit through direct network externalities, then demand will significantly increase and generate substantial marginal benefit through direct network externalities. Thus, demand will neither be well-defined nor stable. The following assumption bounds the size of direct network externalities to rule out such situations.

[5] If the market is not fully covered, then the case referred to as "local monopoly" occurs. Theorem 3 shows that the result derived in Theorem 1 holds in the "local monopoly" case.

***Assumption* 1.**

$$\frac{2}{(2\delta+\Delta)t}\ v'(0)<1$$

For later use, $A\equiv(2\delta+\Delta)2t$. Then, Assumption 1 can be written as $(4/A)\,v'(0)<1$. The following lemma proves useful.

***Lemma* 1.**

(i) Downward sloping:

$$\frac{\partial D_{1i}^{B}}{\partial p_i^{B}}<\frac{\partial D_{0i}^{B}}{\partial p_i^{B}}<0 \text{ for } i=1,2 \text{ and } i\neq j.$$

(ii) Positive cross elasticity:

$$\frac{\partial D_{1j}^{B}}{\partial p_i^{B}}>\frac{\partial D_{0j}^{B}}{\partial p_i^{B}}>0 \text{ for } i=1,2 \text{ and } i\neq j.$$

(iii) Local concavity: Under symmetric prices

$$p_1^B=p_2^B,\ \frac{\partial^2 D_{1i}^{B}}{\partial p_i^{B2}}\leq 0$$

for $i=1,2$. *This condition implies the concavity of* $\log D_{1i}^B$ *under symmetric prices.*[6]

Proof. See Appendix 3. □

With regard to seller-side, a seller has the following options: platforms 1, 2, 0_1, and 0_2. The gross-per-transaction surplus of a seller who is joining platform $i\,(i=1,2)$ is b^S. Therefore, given p_1^S and p_2^S, a seller located at y and joining platform $i\,(i=1,2)$ has utility

$$b^S-p_i^S-\tau|y-y_i|, \tag{17}$$

[6] In equilibrium analysis, attention will be restricted to a symmetric equilibrium. However, this local concave property is insufficient to ensure that a symmetric equilibrium will be the global maximum, but can ensure that it is the local maximum.

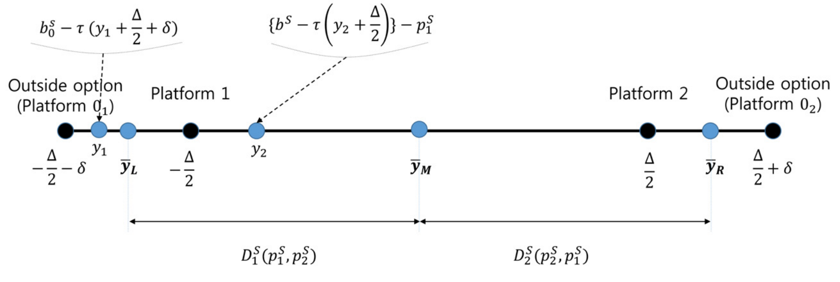

FIGURE 6
DUOPOLY PLATFORMS: SELLER-SIDE

where τ denotes transportation cost and $y_1 \equiv -\Delta/2$, $y_2 \equiv \Delta/2$.

By contrast, if a seller located at y joins platform 0_i ($i=1, 2$), then his/her utility will be

$$b_0^S - \tau|y - y_{0_i}|, \tag{18}$$

where $y_{0_1} = -(\Delta/2) - \delta$ and $y_{0_2} = (\Delta/2) + \delta$. Similar to the case of buyers, sellers located on $[-(\Delta/2) - \delta, 0)$ will never choose platform 0_2, whereas those on $(0, (\Delta/2) + \delta]$ will never join platform 0_1. b^S and b_0^S are assumed to be sufficiently large, and thus, the seller-side market is fully covered.

The net surplus of a seller located at y_1 who chooses platform 0_1 is depicted in the upper left of Figure 6, whereas that of a seller at y_2 who chooses platform 1 is illustrated in the upper right. As shown in Figure 6, given p_1^S and p_2^S, we can derive the seller-side demand for platform i ($i=1, 2$) by following a logic similar to that of buyers, which is given as follows:

$$D_i^S(p_i^S, p_j^S) = \frac{1}{(2\delta + \Delta)} \frac{1}{2\tau} [(p_j^S - 2p_i^B) + (b^S - b_0^S) + \tau(\delta + \Delta)]. \tag{19}$$

Similar to that in the monopoly case, the manner in which indirect network effect occurs is illustrated by defining net per-transaction surpluses on each side as $V_i^B(p_i^B, p_j^B) \equiv \int_{p_i^B}^{+\infty} D_{1i}^B(t, p_j^B)dt$ and $V_i^S(p_i^S, p_j^S) \equiv \int_{p_i^S}^{+\infty} D_i^S(t, p_j^S)dt$. Then, the average net surpluses on each side is given by

$$W_i^B(p_i^B, p_j^B, p_i^S, p_j^S) \equiv V_i^B(p_i^B, p_j^B) \cdot D_i^S(p_i^S, p_j^S), \tag{20}$$

$$W_i^S(p_i^S, p_j^S, p_i^B, p_j^B) \equiv V_i^S(p_i^S, p_j^S) \cdot D_{1i}^B(p_i^B, p_j^B). \tag{21}$$

An increase in demand on one side leads to an increase in the average net surplus on the other side. This situation reflects indirect network effect among end users.

B. Pricing Decision

Platform $i\,(i=1,2)$ attempts to maximize its profit by choosing p_i^B and p_i^S appropriately. The total profit of platform i is determined as follows:

$$\pi_i=(p_i^B+p_i^S-c)D_{1i}^B(p_i^B,p_j^B)D_i^S(p_i^S,p_j^S).\quad (i\neq j).$$

This maximization problem is characterized by FOCs as follows:

$$\frac{\partial(\log\pi_i)}{\partial p_i^B}=\frac{1}{p_i^B+p_i^S-c}+\frac{1}{D_{1i}^B(p_i^B,p_j^B)}\frac{\partial D_{1i}^B(p_i^B,p_j^B)}{\partial p_i^B}=0,$$

$$\frac{\partial(\log\pi_i)}{\partial p_i^S}=\frac{1}{p_i^B+p_i^S-c}+\frac{1}{D_i^S(p_i^S,p_j^S)}\frac{\partial D_i^S(p_i^S,p_j^S)}{\partial p_i^S}=0,$$

where $i\neq j$.

Attention is limited to a symmetric equilibrium. Deriving an explicit expression for symmetric equilibrium prices for the hypothetical situation without direct network effect is a straightforward process that involves plugging in $v(.)\equiv 0$ in Equations (43), (44), and (45) in Appendix 4. The existence of symmetric equilibrium for the proposed duopoly model with direct network effect can be deduced using Theorems 2 and 3.

Symmetric equilibrium prices are denoted as $\hat{P}^B=\hat{p}_1^B=\hat{p}_2^B$ and $\hat{P}^S=\hat{p}_1^S=\hat{p}_2^S$.[7] Then, equilibrium conditions are pinned down to

$$\hat{P}^B+\hat{P}^S-c=-\frac{\hat{D}_{1i}^B}{\partial\hat{D}_{1i}^B/\partial p_i^B},\tag{22}$$

$$\hat{P}^B+\hat{P}^S-c=-\frac{\hat{D}_i^S}{\partial\hat{D}_i^S/\partial p_i^S},\tag{23}$$

where $\hat{D}_{1i}^B\equiv D_{1i}^B(\hat{P}^B,\hat{P}^B)$ and $\hat{D}_i^S\equiv D_i^S(\hat{P}^S,\hat{P}^S)$.

The elasticity of buyer-side demand for platform $i\,(i=1,2)$ is defined

[7] A capital letter P is used to denote symmetric prices in a duopoly case to avoid confusion with prices in a monopoly case.

as $\varepsilon_{1i}^{B}(p_i^B, p_j^B) \equiv -(p_i^B)/(D_{1i}^B)\ (\partial D_{1i}^B)/(\partial p_i^B)$ and that of seller-side demand as $\varepsilon_i^S(p_i^S, p_j^S) \equiv -(p_i^S)/(D_i^S)\ (\partial D_i^S)/(\partial p_i^S)$.[8] Then, we obtain the following expression that characterizes symmetric equilibrium under duopoly platforms:[9]

$$\hat{P}^B + \hat{P}^S - c = \hat{P}^B / \varepsilon_{1i}^B(\hat{P}^B, \hat{P}^B), \tag{24}$$

$$\hat{P}^B + \hat{P}^S - c = \hat{P}^S / \varepsilon_i^S(\hat{P}^S, \hat{P}^S). \tag{25}$$

Equations (24) and (25) are necessary and sufficient conditions for symmetric equilibrium based on the local concavity of buyer-side demand. For notational simplicity, let $\hat{\varepsilon}_1^B \equiv \varepsilon_{1i}^B(\hat{P}^B, \hat{P}^B)$ and $\hat{\varepsilon}^S \equiv \varepsilon_i^S(\hat{P}^S, \hat{P}^S)$. Assuming $\hat{\varepsilon}_1 \equiv \hat{\varepsilon}_1^B + \hat{\varepsilon}^S > 1$, the following proposition can be directly obtained.

***Proposition* 2.**

Under symmetric equilibrium,

(i) The total price of each platform $\hat{P} \equiv \hat{P}^B + \hat{P}^S$ is given by the standard Lerner formula for total volume elasticity as follows:

$$\frac{\hat{P} - c}{\hat{P}} = \frac{1}{\hat{\varepsilon}_1}. \tag{26}$$

(ii) The price structure is given by the ratio of elasticities as follows:

$$\frac{\hat{P}^B}{\hat{\varepsilon}_1^B} = \frac{\hat{P}^S}{\hat{\varepsilon}^S}. \tag{27}$$

An appropriate expression for equilibrium prices is provided in Appendix 4.

C. With Direct Network Effect versus Without Direct Network Effectunder Duopoly

Similar to that in Section II, "benchmark" is defined as the situation where only cross-network effect exists. Assume $v(z) \equiv 0,\ \forall z \in [0, 1]$.

[8] The exact formula for this expression is $\varepsilon_i^S(p_i^S, p_j^S) = (2p_i^S)/((p_j^S - 2p_i^S) + (b^S - b_0^S) + \tau(\delta + \Delta))$.

[9] Note that under symmetric equilibrium prices $\hat{P}^B$ and $\hat{P}^S$, $\varepsilon_{11}^B(\hat{P}^B, \hat{P}^B) = \varepsilon_{12}^B(\hat{P}^B, \hat{P}^B)$ and $\varepsilon_1^S(\hat{P}^S, \hat{P}^S) = \varepsilon_2^S(\hat{P}^S, \hat{P}^S)$.

Then, $\Omega = 1/2$, and thus, symmetric optimal prices chosen by platforms are directly obtained using the expression of equilibrium prices provided in Appendix 4.

$$\tilde{P}^B \equiv \tilde{p}_1^B = \tilde{p}_2^B = \frac{1}{5}\left[3\{(b^B - b_0^B) + t(\delta+\Delta)\} - 2\{(b^S - b_0^S) + \tau(\delta+\Delta)\} + 2c\right] \quad (28)$$

$$\tilde{P}^S \equiv \tilde{p}_1^S = \tilde{p}_2^S = \frac{1}{5}\left[3\{(b^S - b_0^S) + \tau(\delta+\Delta)\} - 2\{(b^B - b_0^B) + t(\delta+\Delta)\} + 2c\right] \quad (29)$$

$$\tilde{P} \equiv \tilde{P}^B + \tilde{P}^S = \frac{1}{5}\left[\{(b^B - b_0^B) + t(\delta+\Delta)\} + \{(b^S - b_0^S) + \tau(\delta+\Delta)\} + 4c\right] \quad (30)$$

For later use, the elasticity of buyer-side demand for platform i in the benchmark case is defined as

$$\varepsilon_{0i}^B\,(p_i^B, p_j^B) \equiv -\frac{p_i^B}{D_{0i}^B}\frac{\partial D_{0i}^B}{\partial p_i^B} = \frac{2p_i^B}{(p_j^B - 2p_i^B) + (b^B - b_0^B) + t(\delta+\Delta)};$$

and $\varepsilon_{0i}^B(\tilde{P}^B, \tilde{P}^B)$, $\varepsilon_{1i}^B(\tilde{P}^B, \tilde{P}^B)$, and $\varepsilon_i^S(\tilde{P}^S, \tilde{P}^S)$ are denoted as $\tilde{\varepsilon}_0^B$, $\tilde{\varepsilon}_1^B$, and $\tilde{\varepsilon}^S$, respectively.

In the case under direct network effect, symmetric equilibrium prices $\hat{P}^B$ and $\hat{P}^S$ are characterized by Proposition 3. Theorem 2 shows that under competition, the side with direct network effect can receive a discount if sufficient network externalities exist. Recall that $\tilde{\varepsilon}_0^B$, $\tilde{\varepsilon}_1^B$, and $\tilde{\varepsilon}^S$ are elasticities evaluated at benchmark equilibrium prices $\tilde{P}^B$ and $\tilde{P}^S$, whereas $\hat{\varepsilon}_1^B$ and $\hat{\varepsilon}^S$ are evaluated at equilibrium prices under direct network effect $\hat{P}^B$ and $\hat{P}^S$. Similarly, "$\tilde{D}_{0i}^B$, $\tilde{D}_{1i}^B$, and $\tilde{D}_i^S$" and "$\hat{D}_{1i}^B$ and $\hat{D}_i^S$" are corresponding demands evaluated at "$\tilde{P}^B$ and $\tilde{P}^S$" and "$\hat{P}^B$ and $\hat{P}^S$", respectively. $P \equiv P^B + P^S$.

Theorem 2.

Unique symmetric solutions $\hat{P}^B$ and $\hat{P}^S$ exist for solving Equations (26) and (27), respectively.

Moreover, the following expressions hold:

(i) If $\hat{\varepsilon}_1^B > \hat{\varepsilon}_0^B$, then $\hat{P}^B < \tilde{P}^B$, $\hat{P}^S > \tilde{P}^S$, and $\hat{P} < \tilde{P}$.
(ii) If $\hat{\varepsilon}_1^B = \hat{\varepsilon}_0^B$, then $\hat{P}^B = \tilde{P}^B$, $\hat{P}^S = \tilde{P}^S$, and $\hat{P} = \tilde{P}$.
(iii) If $\hat{\varepsilon}_1^B < \hat{\varepsilon}_0^B$, then $\hat{P}^B > \tilde{P}^B$, $\hat{P}^S < \tilde{P}^S$, and $\hat{P} > \tilde{P}$.

The necessary sufficient condition for (i) to occur is as follows: the marginal utility generated by direct network effect diminishes slowly such that $avg(v(\tilde{D}_{1i}^{B}))\equiv(V(\tilde{D}_{1i}^{B}))/(\tilde{D}_{1i}^{B})$ is close to $v'(\tilde{D}_{1i}^{B})$. *The exact condition is given as follows:*[10]

$$A-\frac{(2A-6v'(\tilde{D}_{1i}^{B}))}{(2A-3v'(\tilde{D}_{1i}^{B}))}(A-v'(\tilde{D}_{1i}^{B}))>avg(v(\tilde{D}_{1i}^{B})). \tag{31}$$

As a special case, if the marginal utility generated by direct network effect does not diminish (i.e., $v''(z)=0,\ \forall z\in[0, 1]$), *then (i) occurs.*

Proof. See Appendix 5. □

Before investigating the implication of Theorem 2, note that the model considered in this section subsumes the monopoly platform case. Remember that full coverage has been assumed for illustrative simplicity. Suppose that b^B and b^S are insuffciently high. Then, as illustrated in Figure 7, buyers and sellers located near the center of the line $[-(\Delta/2)-\delta,\ \delta+(\Delta/2)]$ may choose to join neither platforms 1 nor 2. This scenario indicates that the market is separated and each platform exercises market power in each region. In this study, this case is referred to as "local monopoly." Theorem 3 shows that the result derived in Theorem 1 holds under the "local monopoly" situation in the model considered in this section.

Theorem 3.

Let the symmetric equilibrium prices for the local monopoly case be $\hat{P}^{B^*}$ *and* $\hat{P}^{S^*}$. *The corresponding buyer-side demand is defined as* $D_{1i}^{B^*}(P^B)$. *Moreover, let the symmetric equilibrium prices for the local monopoly case in the benchmark model (i.e., without direct network effect) be* $\tilde{P}^{B^*}$ *and* $\tilde{P}^{S^*}$. *Then,*

(i) equilibrium prices $\hat{P}^{B^*}$ and $\hat{P}^{S^*}$ *are uniquely determined.*

(ii) Moreover, $\hat{P}^{B^*}\geq\tilde{P}^{B^*}$, $\hat{P}^{S^*}\leq\tilde{P}^{S^*}$, *and* $\hat{P}\geq\tilde{P}$ *hold, where equalities are satisfied if and only if*

[10] Note that $avg(v(\tilde{D}_{1i}^{B}))\geq v'(\tilde{D}_{1i}^{B})$ always holds because of the (weak) concavity of $v(\cdot)$. Moreover, direct calculation shows that $A-((2A-6v'(\tilde{D}_{1i}^{B}))/((2A-3v'(\tilde{D}_{1i}^{B}))(A-v'(\tilde{D}_{1i}^{B}))>v'(\tilde{D}_{1i}^{B})$ always holds. Thus, condition (31) provides the upper and lower bounds of $avg(v(\tilde{D}_{1i}^{B}))$, which ensure the occurrence of (*i*). In Appendix 5, this condition is demonstrated as necessary and sufficient for (*i*) to occur.

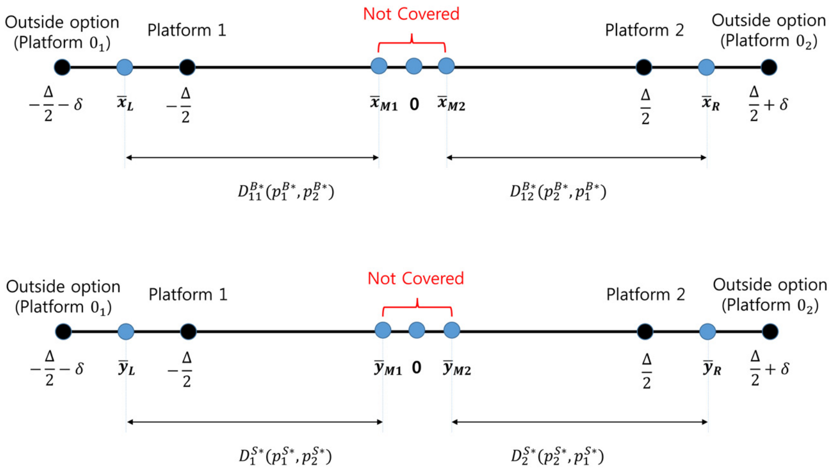

FIGURE 7
LOCAL MONOPOLY

$$\frac{v(D_{11}^{B*}(\tilde{P}^{B*}))}{D_{11}^{B*}(\tilde{P}^{B*})} = v'(D_{11}^{B*}(\tilde{P}^{B*})).$$

Proof. See Appendix 4. □

Similar to that in the monopoly case, demand-augmenting effect and demand-sensitizing effect coexist in a duopoly framework. However, unlike in the monopoly case, demand-sensitizing effect can dominate demand augmenting effect, thereby inducing competing platforms to provide discount to the buyer-side while penalizing the seller-side. Intuitively, introducting platform competition into buyer-side limits demand-augmenting effect because competing platforms should split the total buyer-side demand. Consider a case in which δ is extremely close to 0. In this case, competing platforms 1 and 2 nearly partition buyer-side. In such extreme case, a marginal reduction of t (increase in competition) augments a negligible amount of demand, which in turn suggests that a marginal increase in demand that is purely attributed to direct network effect is also negligible (*i.e.*, negligible demand-augmenting effect).

By contrast, demand sensitizing effect can prevail. Suppose platform i increases buyer-side price. Then, buyer-side demand for platform i decreases more severely in a competitive environment than in a less competitive environment. Evidently, loss of demand will be maximized if $v'(z) \equiv \eta$, $\forall z \in [0, 1]$ (*i.e.*, in the case where the marginal utility generated by direct network effect does not increase as buyer-side demand for platform i decreases). However, if the marginal utility generated by direct network effect diminishes rapidly, then the reduction in demand caused by an increase in p_i^B will be alleviated because the marginal effect in such scenario is stronger with lower demand, which counters the decreasing pressure of demand. Condition (31) ensures that demand-sensitizing effect dominates demand-augmenting effect.

IV. Comparative Statistics

In this section, comparative statistics are provided under the assumption of linear direct network effect (*i.e.*, $v(z) \equiv \eta z$, $\forall z \in [0, 1]$). As shown in Theorem 2, introduction of direct network effect in this case reduces buyer-side price and increases seller-side price, while decreasing overall price. Though the assumption of linear direct network effect seems restrictive, the results provided in this section are preserved to the case where we allow moderate degree of diminishing marginal utility generated by direct network effect, because of continuity.

Under linear direct network effect, we can derive symmetric equilibrium prices explicitly using Equation (49) in Appendix 5, as shown in Appendix 7.

***Theorem* 4.**

Assume that a linear direct network effect occurs. As the magnitude of direct network effect η *increases, buyer-side price* $\hat{P}^B$ *decreases, seller-side price* $\hat{P}^S$ *increases, and overall price* $\hat{P}$ *decreases.*

Proof. See Appendix 7. □

***Theorem* 5.**

Assume that a linear direct network effect occurs. As the market power of the platforms increases in the seller-side (i.e., $\tau\uparrow$*), buyer-side price* $\hat{P}^B$ *decreases, seller-side price* $\hat{P}^S$ *increases, and overall price* $\hat{P}$ *decreases.*

Furthermore, given $\eta > 0$*, the price gap between the benchmark and the case under direct network effect (i.e.,* $\tilde{P}^B - \hat{P}^B$*) increases as* τ *goes up.*

Proof. See Appendix 8. □

Theorem 5 indicates that even under direct network effect, the introduction of competition among platforms in the seller-side relieves the incentive of each platform to lower buyer-side price $\hat{P}^B$ (*i.e.*, $\tau \downarrow \Rightarrow \hat{P}^B \uparrow$).

***Theorem* 6.**

Assume that a linear direct network effect occurs. As the market power of the platforms increases on the buyer-side (i.e., $t\uparrow$), buyer-side price $\hat{P}^B$ increases, seller-side price $\hat{P}^S$ decreases, and overall price $\hat{P}$ increases.

However, whether the price gap between the benchmark and the case under direct network effect (i.e., $\tilde{P}^B - \hat{P}^B$) increases as t goes up remains unclear.

Proof. See Appendix 9. □

Although the sign of $(\partial/\partial t)(\tilde{P}^B - \hat{P}^B)$ remains ambiguous, numerical simulations indicates that for nearly all parameter ranges, $(\partial \hat{P}^B)/\partial t > (\partial \tilde{P}^B)/\partial t$ occurs. Only for sufficiently small values of both $(b^B - b_0^B) + (b^S - b_0^S) - c$ and η can $(\partial \hat{P}^B)/\partial t < (\partial \tilde{P}^B)/\partial t$ occur. This result indicates that under sufficient direct network effect (*i.e.*, sufficiently large η), introducing platform competition into the buyer-side can strengthen the incentive of each platform to lower buyer-side price $\hat{P}^B$ (*i.e.*, $t\downarrow \Rightarrow \hat{P}^B \downarrow$). This result is in line with Theorems 1 and 3, given that monopoly power on the buyer-side weakens demand-sensitizing effect relative to demand-augmenting effect, thereby resulting in an upward pressure on buyer-side price.

V. Conclusion

The optimal pricing strategy of monopoly/duopoly platform(s) where one side of a group is under direct network effect are investigated. In contrast to the monopoly platform framework where demand-augmenting effect dominates, either demand-augmenting effect or demand-sensitizing effect can dominate in the duopoly framework. In particular, if the marginal utility generated by direct network effect diminishes sufficiently slowly as buyer-side demand increases, then demand-sensitizing effect dominates, which induces competing platforms to lower buyer-side price and increase seller-side price. In this study, the sum of both prices decreases, which is in line with the pricing convention of platforms under the competitive environment illustrated in the introduction. These real-world platforms typically charge low fees (or even allow free usage of platform services) to customers and charge high fees to content providers. Moreover, the result implies that even in the case where platforms charge seemingly excessively low prices to buyers, these pricing decisions may reflect strong direct network externalities instead of anticompetitive practices, thereby requiring close scrutiny.

In the special case under linear direct network effect ($v(z)\equiv\eta z$, $\forall z\in[0, 1]$), demand sensitizing effect dominates. Under this environment, the behaviour of competing platforms is strengthened as the magnitude of direct network effect increases. That is, buyer-side price decreases, whereas seller-side price increases. In addition, a stronger competition among platforms in the seller-side implies incentive of each platform to lower buyer-side price is reduced. By contrast, a stronger competition among platforms in the buyer-side implies incentive of each platform to provide a discount to the buyer-side is strengthened. Thus, the effect of introducing direct network effect on the equilibrium buyer-side price depends on the relative competitiveness of each side of the market.

(Received 27 May 2015; Revised 9 June 2016 ; Accepted 11 July 2016)

Appendix

A. Appendix 1

Equilibrium prices of the monopoly model

The equilibrium prices in Proposition 1 can be explicitly derived by substituting the definition of elasticities into Equations (8) and (9). Let $\hat{D}_1^B\equiv D_1^B(\hat{p}^B)$. The equilibrium prices are

$$\hat{p}^B=\frac{1}{3}[2\{\bar{b}^B+v(\hat{D}_1^B)-\hat{D}_1^B\cdot v'(\hat{D}_1^B)\}-\bar{b}^S+c], \tag{32}$$

$$\hat{p}^S=\frac{1}{3}[2\bar{b}^S-\{\bar{b}^B+v(\hat{D}_1^B)-\hat{D}_1^B\cdot v'(\hat{D}_1^B)\}+c], \tag{33}$$

$$\hat{p}=\frac{1}{3}[\{\bar{b}^B+v(\hat{D}_1^B)-\hat{D}_1^B\cdot v'(\hat{D}_1^B)\}+\bar{b}^S+c]. \tag{34}$$

B. Appendix 2

Alternative proof of *Theorem 1*

The logic of the proof of *Theorem 2* is analogous to that provided herein.

Proof. To compare $\hat{p}^B$ and $\hat{p}^S$ with the benchmark $\tilde{p}^B$ and $\tilde{p}^S$, note that the relation $p^B+p^S-c=\bar{b}^S-p^S$ holds in both cases in the equilibrium. Thus, $p^S=(1/2)\,[\bar{b}^S+c-p^B]$ in both cases. By substituting this expression into Equations (6) and (7), we obtain the following equations, respectively:

$$\bar{b}^S-p^S=\frac{1}{2}\,(\bar{b}^S-c+p^B)=D_1^B\,(p^B)[\bar{b}^B-v'(D_1^B\,(p^B))],$$

$$\bar{b}^S-p^S=\frac{1}{2}\,(\bar{b}^S-c+p^B)=\bar{b}^B-p^B.$$

The first expression characterizes the equilibrium of the case under direct and indirect network effects, whereas the second characterizes the equilibrium of the benchmark case. Therefore, based on the definitions of $\hat{p}^B$, $\hat{p}^S$ and $\tilde{p}^B$, $\tilde{p}^S$,

$$\bar{b}^S-\hat{p}^S=\frac{1}{2}\,(\bar{b}^S-c+\hat{p}^B)=D_1^B\,(\hat{p}^B)[\bar{b}^B-v'(D_1^B\,(\hat{p}^B))], \tag{35}$$

$$\bar{b}^S-\tilde{p}^S=\frac{1}{2}\,(\bar{b}^S-c+\tilde{p}^B)=\bar{b}^B-\tilde{p}^B. \tag{36}$$

Before proceeding, note that $p^B<\bar{b}^B$,[11] $D_1^B\,(p^B)>D_0^B\,(p^B)$, and $\bar{b}^B-v'(D_1^B\,(p^B))<\bar{b}^B$, where $D_0^B\,(p^B)\cdot\bar{b}^B\equiv\bar{b}^B-p^B$. Thus, at first glance, both $D_1^B\,(\tilde{p}^B)[\bar{b}^B-v'(D_1^B\,(\tilde{p}^B))]<\bar{b}^B-\tilde{p}^B$ and $D_1^B\,(\tilde{p}^B)[\bar{b}^B-v'(D_1^B\,(\tilde{p}^B))]\geq\bar{b}^B-\tilde{p}^B$ appear possible. However, the former turns out to be impossible.

To demontrate this situation, note that

$$D_1^B\,(\tilde{p}^B)[\bar{b}^B-v'(D_1^B\,(\tilde{p}^B))]=\frac{\bar{b}^B-\tilde{p}^B+v(D_1^B\,(\tilde{p}^B))}{\bar{b}^B}\,[\bar{b}^B-v'(\mathrm{D}_1^B\,(\tilde{p}^B))]$$

[11] Note that $\tilde{p}^B=(1/3)[2\bar{b}^B-\bar{b}^S+c]<\bar{b}^B$ for a moderate value of c.

$$=\{\bar{b}^B-\tilde{p}^B+v(D_1^B(\tilde{p}^B))\}-D_1^B(\tilde{p}^B)v'(D_1^B(\tilde{p}^B)).$$

Thus, $D_1^B(\tilde{p}^B)[\bar{b}^B-v'(D_1^B(\tilde{p}^B))]\geq\bar{b}^B-\tilde{p}^B$ is equivalent to

$$\frac{v(D_1^B(\tilde{p}^B))}{D_1^B(\tilde{p}^B)}\geq v'(D_1^B(\tilde{p}^B)),$$

which always holds based on the assumption $v''(z)\leq 0\ \ \forall z\in[0, 1]$.

Hence, we obtain

$$\frac{1}{2}(\bar{b}^S-c+\tilde{p}^B)\leq D_1^B(\tilde{p}^B)[\bar{b}^B-v'(D_1^B(\tilde{p}^B))]. \qquad (37)$$

Evidently, the left side of the preceding equation is an increasing function of p^B. Note that

$$\frac{d[D_1^B(p^B)\{\bar{b}^B-v'(D_1^B(p^B))\}]}{dp^B}=\frac{dD_1^B(p^B)}{dp^B}\cdot[\{\bar{b}^B-v'(D_1^B(p^B))\}-v''(D_1^B(p^B))\cdot D_1^B(p^B)]<0$$

via Remark 1.(i). That is, the right side of the preceding equation is a decreasing function of p^B.

Therefore, if the inequality in Equation (37) is strict, then a unique price $\hat{p}^B$ higher than $\tilde{p}^B$ exists, such that $(1/2)(\bar{b}^S-c+\hat{p}^B)=D_1^B(\hat{p}^B)[\bar{b}^B-v'(D_1^B(\hat{p}^B))]$ holds. This condition suggests $\hat{p}^S=(1/2)[\bar{b}^S+c-\hat{p}^B]<(1/2)[\bar{b}^S+c-\tilde{p}^B]=\tilde{p}^B$ and $\hat{p}=(1/2)[\bar{b}^S+c+\hat{p}^B]>(1/2)[\bar{b}^S+c+\tilde{p}^B]$.

Moreover, $\hat{p}^B=\tilde{p}^B$, $\hat{p}^S=\tilde{p}^S$, and $\hat{p}=\tilde{p}$ hold if and only if Equation (37) holds in equality, and if and only if

$$\frac{v(D_1^B(\tilde{p}^B))}{D_1^B(\tilde{p}^B)}=v'(D_1^B(\tilde{p}^B)).$$ □

C. Appendix 3

Lemma 1.

Proof. Without losing generality, let $i=1$ and $j=2$. First, we can directly calculate

$$\frac{\partial D_{01}^B}{\partial p_1^B}=-\frac{2}{A} \text{ and } \frac{\partial D_{02}^B}{\partial p_1^B}=\frac{1}{A} \text{ from Equation (16).}$$

Then, by differentiating D_{11}^{B} in Equation (14) with respect to p_1^B, we obtain

$$\frac{\partial D_{11}^{B}}{\partial p_1^B}=\frac{1}{A}\,[2v'(D_{11}^{B})\frac{\partial D_{11}^{B}}{\partial p_1^B}-v'(D_{12}^{B})\frac{\partial D_{12}^{B}}{\partial p_1^B}-2].$$

This equation can be rewritten as

$$[1-\frac{1}{A}\;2v'(D_{11}^{B})]\frac{\partial D_{11}^{B}}{\partial p_1^B}+\frac{1}{A}\,v'(D_{12}^{B})\frac{\partial D_{12}^{B}}{\partial p_1^B}=\frac{\partial D_{01}^{B}}{\partial p_1^B}. \tag{38}$$

Similarly, by differentiating D_{12}^{B} in Equation (15) with respect to p_1^B, we can show that

$$[1-\frac{1}{A}\;2v'(D_{12}^{B})]\frac{\partial D_{12}^{B}}{\partial p_1^B}+\frac{1}{A}\,v'(D_{11}^{B})\frac{\partial D_{11}^{B}}{\partial p_1^B}=\frac{\partial D_{02}^{B}}{\partial p_1^B}. \tag{39}$$

We can observe that Equations (38) and (39) form simultaneous equations in which $(\partial D_{11}^{B})/(\partial p_1^B)$ and $(\partial D_{12}^{B})/(\partial p_1^B)$ are unknowns. By solving these equations, we obtain

$$\frac{\partial D_{11}^{B}}{\partial p_1^B}=\frac{\{1-\frac{2}{A}\;v'(D_{12}^{B})\}\;\frac{\partial D_{01}^{B}}{\partial p_1^B}-\frac{v'(D_{12}^{B})}{A^2}}{\{1-\frac{2}{A}\,v'(D_{11}^{B})\}\{1-\frac{2}{A}\,v'(D_{12}^{B})\}-\frac{v'(D_{11}^{B})v'(D_{12}^{B})'}{A^2}}, \tag{40}$$

$$\frac{\partial D_{12}^{B}}{\partial p_1^B}=\frac{\frac{\partial D_{02}^{B}}{\partial p_1^B}}{\{1-\frac{2}{A}\,v'(D_{11}^{B})\}\{1-\frac{2}{A}\,v'(D_{12}^{B})\}-\frac{v'(D_{11}^{B})v'(D_{12}^{B})}{A^2}}. \tag{41}$$

Consider Equation (40). The following relations hold based on Assumption 1:

$$\begin{aligned}0<1&-\frac{2}{A}\,\{v'(D_{11}^{B})+v'(D_{12}^{B})\}\\ &<1-\frac{2}{A}\,\{v'(D_{11}^{B})+v'(D_{12}^{B})\}+\frac{3}{A^2}\,v'(D_{11}^{B})v'(D_{12}^{B})\end{aligned}$$

$$=\{1-\frac{2}{A}v'(D_{11}^{B})\}\{1-\frac{2}{A}v'(D_{12}^{B})\}-\frac{v'(D_{11}^{B})v'(D_{12}^{B})}{A^{2}}.$$

By dividing the denominator and the numerator of Equation (40) by $\{1-(2/A)\,v'(D_{12}^{B})\}(>0)$, we obtain

$$\frac{\partial D_{11}^{B}}{\partial p_{1}^{B}}=\frac{\dfrac{\partial D_{01}^{B}}{\partial p_{1}^{B}}-\dfrac{\frac{v'(D_{12}^{B})}{A^{2}}}{\{1-\frac{2}{A}v'(D_{12}^{B})\}}}{\{1-\frac{2}{A}v'(D_{11}^{B})\}-\dfrac{\frac{v'(D_{11}^{B})v'(D_{12}^{B})}{A^{2}}}{\{1-\frac{2}{A}v'(D_{12}^{B})\}}}. \tag{42}$$

Note that the denominator of Equation (42) is positive (given that a positive value divided by a positive term is positive) and smaller than 1 (given that both

$$\frac{2}{A}v'(D_{11}^{B})\ \text{and}\ \frac{\frac{v'(D_{11}^{B})v'(D_{12}^{B})}{A^{2}}}{\{1-\frac{2}{A}v'(D_{12}^{B})\}}$$

are positive). Thus, the following relations hold:

$$\frac{\partial D_{11}^{B}}{\partial p_{1}^{B}}=\frac{\dfrac{\partial D_{01}^{B}}{\partial p_{1}^{B}}-\dfrac{\frac{v'(D_{12}^{B})}{A^{2}}}{\{1-\frac{2}{A}v'(D_{12}^{B})\}}}{\{1-\frac{2}{A}v'(D_{11}^{B})\}-\dfrac{\frac{v'(D_{11}^{B})v'(D_{12}^{B})}{A^{2}}}{\{1-\frac{2}{A}v'(D_{12}^{B})\}}}$$

$$< \frac{\frac{\partial \mathrm{D}_{01}^B}{\partial p_1^B}}{\{1-\frac{2}{A}v'(\mathrm{D}_{11}^B)\}-\frac{\frac{v'(\mathrm{D}_{11}^B)v'(D_{12}^B)}{A^2}}{\{1-\frac{2}{A}v'(D_{12}^B)\}}} < \frac{\partial \mathrm{D}_{01}^B}{\partial p_1^B}.$$

The first part of Lemma 1 is completed.

Consider Equation (41). The following relations hold based on Assumption 1.

$$\begin{aligned}
0 &< 1-\frac{2}{A}\{v'(D_{11}^B)+v'(D_{12}^B)\} \\
&< 1-\frac{2}{A}\{v'(D_{11}^B)+v'(D_{12}^B)\}+\frac{3}{A^2}v'(D_{11}^B)v'(D_{12}^B) \\
&= \{1-\frac{2}{A}v'(D_{11}^B)\}\{1-\frac{2}{A}v'(D_{12}^B)\}-\frac{(v'(D_{11}^B)v'(D_{12}^B)}{\mathrm{A}^2}) \\
&< \{1-\frac{2}{A}v'(D_{11}^B)\}\{1-\frac{2}{A}v'(D_{12}^B)\} < 1.
\end{aligned}$$

The denominator of Equation (41) is positive and smaller than 1. Thus,

$$\frac{\partial D_{12}^B}{\partial p_1^B} = \frac{\frac{\partial D_{02}^B}{\partial p_1^B}}{\{1-\frac{2}{A}v'(D_{11}^B)\}\{1-\frac{2}{A}v'(D_{12}^B)\}-\frac{v'(D_{11}^B)v'(D_{12}^B)}{A^2}} > \frac{\partial D_{02}^B}{\partial p_1^B}.$$

The second part of the proof is completed.

Now, the concavity of D_{1i}^B will be shown with respect to p_i^B under symmetric prices $p_1^B = p_2^B$. Without loss, let $i=1$. For notational simplicity, denote the numerator of Equation (40) as H_N and the denominator as H_D. Then,

$$\frac{\partial^2 \mathrm{D}_{11}^B}{\partial p_1^{B2}} = \frac{1}{H_D^2}[H_D \cdot \frac{\partial H_N}{\partial p_1^B} - \frac{\partial H_D}{\partial p_1^B} \cdot H_N].$$

First, note that $H_N < 0$ and $H_D > 0$ have already been established. $(\partial H_N)/(\partial p_1^B) \leq 0$ and $(\partial H_D)/(\partial p_1^B) \leq 0$, which implies that $[H_D \cdot (\partial H_N)/(\partial$

$p_1^B) - (\partial H_D)/(\partial p_1^B)\cdot H_N] \le 0$, and thus, $(\partial^2 D_{11}^B)/(\partial p_1^{B2}) \le 0$.

First, we can directly calculate

$$\frac{\partial H_N}{\partial p_1^B} = \frac{\partial}{\partial p_1^B}(\{1 - \frac{2}{A} v'(D_{12}^B)\} \frac{\partial D_{01}^B}{\partial p_1^B} - \frac{v'(D_{12}^B)}{A^2} = \frac{3}{A^2} v''(D_{12}^B) \frac{\partial D_{12}^B}{\partial p_1^B} \le 0.$$

Second,

$$\begin{aligned}\frac{\partial H_D}{\partial p_1^B} &= \frac{\partial}{\partial p_1^B}(\{1 - \frac{2}{A} v'(D_{11}^B)\}\{1 - \frac{2}{A} v'(D_{12}^B)\} - \frac{v'(D_{11}^B) v'(D_{12}^B)}{A^2} \\ &= \frac{\partial}{\partial p_1^B}(1 - \frac{2}{A}\{v'(D_{11}^B) + v'(D_{12}^B)\} + \frac{3}{A^2} v'(D_{11}^B) v'(D_{12}^B)) \\ &= v''(D_{11}^B) \frac{\partial D_{11}^B}{\partial p_1^B} [-\frac{2}{A} + \frac{3}{A^2} v'(D_{12}^B)] + v''(D_{12}^B) \frac{\partial D_{12}^B}{\partial p_1^B} [-\frac{2}{A} + \frac{3}{A^2} v'(D_{11}^B)] \\ &= v''(D_{11}^B)[-\frac{2}{A} + \frac{3}{A^2} v'(D_{11}^B)] (\frac{\partial D_{11}^B}{\partial p_1^B} + \frac{\partial D_{12}^B}{\partial p_1^B}).\end{aligned}$$

Given that we are considering symmetric prices, $D_{11}^B = D_{12}^B$ holds, thereby implying that $v'(D_{11}^B) = v'(D_{12}^B)$ and $v''(D_{11}^B) = v''(D_{12}^B)$, from which the last equality follows. Assumption 1 implies that $[-(2/A) + (3/A^2) v'(D_{11}^B)] < 0$. By summing (40) and (41), we obtain $((\partial D_{11}^B)/(\partial p_1^B) + (\partial D_{12}^B)/(\partial p_1^B)) = (1/B)[-(1/A) + (3/A^2) v'(D_{12}^B)] < 0$. Given that $v''(D_{11}^B) \le 0$, we conclude that $(\partial H_D)/(\partial p_1^B) \le 0$. The third part of Lemma 1 is thus completed. □

D. Appendix 4

Equilibrium prices of duopoly model

Let $\hat{D}_1^B \equiv D_{1i}^B(\hat{P}^B, \hat{P}^B), \forall i = 1,2$. Then, the equilibrium prices of the duopoly model can be expressed as follows:

$$\hat{P}^B = \frac{3\Omega}{3\Omega+1}[v(\hat{D}_1^B) + (b^B - b_0^B) + t(\delta + \Delta)] - \frac{1}{3\Omega+1}[(b^S - b_0^S) + \tau(\delta + \Delta)] + \frac{1}{3\Omega+1} c, \tag{43}$$

$$\hat{P}^S = \frac{\Omega+1}{3\Omega+1}[(b^S - b_0^S) + \tau(\delta + \Delta)] - \frac{2\Omega}{3\Omega+1}[v(\hat{D}_1^B) + (b^B - b_0^B) + t(\delta + \Delta)] \tag{44}$$

$$+\frac{2\Omega}{3\Omega+1}c,$$

$$\hat{P}=\frac{\Omega}{3\Omega+1}[v(\hat{D}_1^B)+(b^B-b_0^B)+t(\delta+\Delta)]+\frac{\Omega}{3\Omega+1}[(b^S-b_0^S)+\tau(\delta+\Delta)] \\ +\frac{2\Omega+1}{3\Omega+1}c, \tag{45}$$

where $\Omega\equiv\dfrac{\{1-\frac{2}{A}v'(\hat{D}_1^B)\}^2-\frac{1}{A^2}v'(\hat{D}_1^B)\}^2}{2\{1-\frac{2}{A}v'(\hat{D}_1^B)\}-\frac{1}{A}v'(\hat{D}_1^B)}$.

Proof. Recall that buyer-side demands are given by

$$D_{1i}^B=\frac{1}{A}[2v(D_{1i}^B-v(D_{1j}^B)]+\frac{1}{A}[(p_j^B-2p_i^B)+(b^B-b_0^B)+t(\delta+\Delta)],\ (i\neq j).$$

From Lemma 1(i), the following expression is obtained:

$$\frac{\partial D_{1i}^B}{\partial p_i^B}=-\frac{\{1-\frac{2}{A}v'(D_{1j}^B)\}\frac{2}{A}+\frac{v'(D_{1j}^B)}{A^2}}{\{1-\frac{2}{A}v'(D_{1i}^B)\}\{1-\frac{2}{A}v'(D_{1j}^B)\}-\frac{v'(D_{1i}^B)v'(D_{1j}^B)}{A^2}},\ (i\neq j).$$

Using the preceding expression, FOCs (22) and (23) can be expressed as

$$\hat{P}^B+\hat{P}^S-c=\frac{[\{1-\frac{2}{A}v'(\hat{D}_1^B)\}^2-\frac{1}{A^2}\{v'(\hat{D}_1^B)\}^2][\mathrm{v}(\hat{D}_1^B)+\{-\hat{P}^B+(b^B-b_0^B)+t(\delta+\Delta)\}]}{2\{1-\frac{2}{A}v'(\hat{D}_1^B)\}-\frac{1}{A}v'(\hat{D}_1^B)}, \tag{46}$$

$$\hat{P}^B+\hat{P}^S-c=\frac{-\hat{P}^S+(b^S-b_0^S)+\tau(\delta+\Delta)}{2}, \tag{47}$$

where $\hat{D}_1^B\equiv D_{1i}^B\ (\hat{P},\hat{P}),\forall i=1,2$.

After rearranging Equation (47) into $\hat{P}^S=(1/3)[-2\hat{P}^B+(b^S-b_0^S)+2c+\tau(\delta+\Delta)]$, we substitute this expression into Equation (46), which yields

$$\frac{1}{3}[\hat{P}^B+(b^S-b_0^S)+\tau(\delta+\Delta)-c]=\Omega[v(\hat{D}_1^B)+\{-\hat{P}^B+(b^B-b_0^B)+t(\delta+\Delta)\}], \quad (48)$$

where

$$\Omega\equiv\frac{\{1-\dfrac{2}{A}v'(\hat{D}_1^B)\}^2-\dfrac{1}{A^2}v'(\hat{D}_1^B)\}^2}{2\{1-\dfrac{2}{A}v'(\hat{D}_1^B)\}-\dfrac{1}{A}v'(\hat{D}_1^B)}.$$

Rearranging Equation (48) yields expression Equation (43). The expressions for $\hat{P}^S$ and $\hat{P}$ can also be derived by substituting Equation (43) into Equation (47). □

E. Appendix 5

Theorem 2.

Proof. To compare $\hat{P}^B$ and $\hat{P}^S$ with benchmark $\tilde{P}^B$ and $\tilde{P}^S$, note first that the relation $p_i^B+p_i^S-c=((p_j^S-2p_i^S)+(b^S-b_0^S)+\tau(\delta+\Delta))/2$ holds in both cases in equilibrium. This condition follows directly from the FOC with respect to p_i^S, where $((p_j^S-2p_i^S)+(b^S-b_0^S)+\tau(\delta+\Delta))/2$ is the expression for $-(D^S(p_i^S, p_j^S))/(\partial D^S(p_i^S, p_j^S)/\partial p_i^S)$. Given that we are considering symmetric equilibrium, this relation can be written as $P^B+P^S-c=((P^S-2P^S)+(b^S-b_0^S)+\tau(\delta+\Delta))/2$, or $P^S=(1/3)[-2P^B+(b^S-b_0^S)+2c+\tau(\delta+\Delta)]$ after rearrangement. In this study, $P^B\equiv p_1^B=p_2^B$ and $P^S\equiv p_1^S=p_2^S$ denote symmetric prices. By substituting these symmetric prices into Equation (22), we obtain

$$\frac{1}{3}[\hat{P}^B+(b^S-b_0^S)+\tau(\delta+\Delta)-c]=-\frac{\hat{D}_{1i}^B}{\partial\hat{D}_{1i}^B/\partial p_i^B}, \quad (49)$$

$$\frac{1}{3}[\tilde{P}^B+(\mathrm{b}^S-b_0^S)+\tau(\delta+\Delta)-\mathrm{c}]=\frac{-\tilde{P}^B+(b^B-b_0^B)+t(\delta+\Delta)}{2}, \quad (50)$$

where $(-\tilde{P}^B+(b^B-b_0^B)+t(\delta+\Delta))/2$ is the expression for $-(\tilde{D}_{0i}^B)/(\partial\tilde{D}_{0i}^B/\partial p_i^B)$. The first relation characterizes the equilibrium of the case under both direct and indirect network effects, whereas the second relation characterizes the benchmark case.

Before proceeding, note that $\tilde{D}_{0i}^B < \tilde{D}_{0i}^B + (1/A)\, v(\tilde{D}_{1i}^B) = \tilde{D}_{1i}^B$ based on Equations (14), (15), and (16). Moreover, $A/2 = -1/(\partial \tilde{D}_{0i}^B/\partial p_i^B) > -1/(\partial \tilde{D}_{1i}^B/\partial p_i^B)$ according to Lemma 1(i). Thus, $-(\tilde{D}_{1i}^B)/(\partial \tilde{D}_{1i}^B/\partial p_i^B) < -(\tilde{D}_{0i}^B)/(\partial \tilde{D}_{0i}^B/\partial p_i^B)$ and $-(\tilde{D}_{1i}^B)/(\partial \tilde{D}_{1i}^B/\partial p_i^B) \geq -(\tilde{D}_{0i}^B)/(\partial \tilde{D}_{0i}^B/\partial p_i^B)$ are both possible, depending on the relative size of $\tilde{D}_{1i}^B/\tilde{D}_{0i}$ and $(\partial \tilde{D}_{1i}^B/\partial p_i^B)/(\partial \tilde{D}_{0i}^B/\partial p_i^B)$.

Assume $\tilde{\varepsilon}_1^B > \tilde{\varepsilon}_0^B$. By definition, this relation is equivalent to $-(\tilde{D}_{1i}^B)/(\partial \tilde{D}_{1i}^B/\partial p_i^B) < -(\tilde{D}_{0i}^B)/(\partial \tilde{D}_{0i}^B/\partial p_i^B) (= (-\tilde{P}^B + (b^B - b_0^B) + t(\delta+\Delta))/2)$. Hence,

$$\frac{1}{3}[\tilde{P}^B + (b^S - b_0^S) + \tau(\delta+\Delta) - c] > -\frac{\tilde{D}_{1i}^B}{\partial \tilde{D}_{1i}^B/\partial p_i^B}.$$

The left side of the preceding equation is clearly an increasing function of p^B. The right hand side is a decreasing function of P^B. (Claim 1 below.) This situation establishes that a unique price $\hat{P}^B$ exists, which is strictly smaller than $\tilde{P}^B$, such that $(1/3)[\hat{P}^B + (b^S - b_0^S) + \tau(\delta+\Delta) - c] = -(\hat{D}_{1i}^B)/(\partial \hat{D}_{1i}^B/\partial p_i^B)$. This equation, in turn, suggests $\hat{P}^S = (1/3)[-2\hat{P}^B + (b^S - b_0^S) + 2c + \tau(\delta+\Delta)] > (1/3)[-2\tilde{P}^B + (b^S - b_0^S) + 2c + \tau(\delta+\Delta)] = \tilde{P}^S$ and $\hat{P} = (1/3)[\hat{P}^B + (b^S - b_0^S) + 2c + \tau(\delta+\Delta)] < (1/3)[\tilde{P}^B + (b^S - b_0^S) + 2c + \tau(\delta+\Delta)] = \tilde{P}$.

By reversing the sign, the $\tilde{\varepsilon}_1^B = \tilde{\varepsilon}_0^B$ and $\tilde{\varepsilon}_1^B < \tilde{\varepsilon}_0^B$ cases can be proven.

For the second part of the theorem, symmetric prices are assumed. Then, from Equations (14) and (15),

$$D_{1i}^B = \frac{1}{A - avg(v(D_{1i}^B))}\{-P^B + (b^B - b_0^B) + t(\delta+\Delta)\}. \tag{51}$$

Similarly, the following relation holds under symmetric prices according to Lemma 1.(i):

$$\frac{\partial D_{1i}^B}{\partial p_i^B} = -\frac{2A - 3v'(D_{1i}^B)}{(A - v'(D_{1i}^B))(A - 3v'(D_{1i}^B))}. \tag{52}$$

Thus,

$$-\frac{D_{1i}^B}{\partial D_{1i}^B/\partial p_i^B} = \{\frac{A - v'(D_{1i}^B)}{A - avg(v(D_{1i}^B))}\}\{\frac{A - 3v'(D_{1i}^B)}{2A - 3v'(D_{1i}^B)}\}\{-P^B + (b^B - b_0^B) + t(\delta+\Delta)\}. \tag{53}$$

Given that $-(D_{0i}^B)/(\partial D_{0i}^B/\partial p_i^B) = (1/2)\{-P^B + (b^B - b_0^B) + t(\delta+\Delta)\}$, direct calculation shows that the necessary condition for (i) to occur (*i.e.*, $-(\tilde{D}_{1i}^B)/(\partial\tilde{D}_{1i}^B/\partial p_i^B) < -(\tilde{D}_{0i}^B)/(\partial\tilde{D}_{0i}^B/\partial p_i^B)$) is equivalent to

$$A-\frac{(2A-6v'(\tilde{D}_{1i}^B))}{2A-3v'(\tilde{D}_{1i}^B))}(A-v'(\tilde{D}_{1i}^B))>avg(v(\tilde{D}_{1i}^B)).$$

Moreover,

$$A-\frac{(2\mathrm{A}-6v'(D_{1i}^B))}{(2\mathrm{A}-3v'(D_{1i}^B))}(\mathrm{A}-v'(D_{1i}^B))>v'(D_{1i}^B)$$

always holds. If $v''(z)=0,\ \forall z\in[0,1]$ then $avg(v(D_{1i}^B))\equiv v'(D_{1i}^B)$. The second part of the proof is now completed. □

***Claim* 1.** *$-(D_{1i}^B(P^B,P^B))/(\partial D_{1i}^B(P^B,P^B)/\partial p_i^B)$ is a decreasing function of P^B.*

Proof. Without losing generality, let $i=1$, $j=2$.

$$\frac{d}{dP^B}(D_{11}(P^B,P^B)\cdot\{-\frac{1}{\frac{\partial D_{11}^B(P^B,P^B)}{\partial p_1^B}}\})$$

$$=[\{\frac{\partial D_{11}^B(P^B,P^B)}{\partial p_1^B}+\frac{\partial D_{11}^B(P^B,P^B)}{\partial p_2^B}\}\cdot\{-\frac{1}{\frac{\partial D_{11}^B(P^B,P^B)}{\partial p_1^B}}\}] \tag{54}$$

$$+[D_{11}^B(P^B,P^B)\cdot\{\frac{\partial D_{11}^B(P^B,P^B)}{\partial p_1^B}\}^{-2}\cdot\{\frac{\partial^2 D_{11}^B(P^B,P^B)}{\partial p_1^{B2}}+\frac{\partial^2 D_{11}^B(P^B,P^B)}{\partial p_1^B\,\partial p_2^B}\}].$$

Consider the first brackets in Equation (54). As observe from (41), $(\partial D_{12}^B)/(\partial p_1^B)=(\partial D_{11}^B)/(\partial p_2^B)$ holds because $(\partial D_{02}^B)/(\partial p_1^B)=(\partial D_{01}^B)/(\partial p_2^B)=1/A$. Thus, by summing Equations (40) and (41), and then evaluating the resulting equation at $p_1^B=p_2^B=P^B$, we obtain

$$\frac{\partial D_{11}^B(P^B,P^B)}{\partial p_1^B}+\frac{\partial D_{11}^B(P^B,P^B)}{\partial p_2^B}, \tag{55}$$

$$=\frac{-\frac{1}{A}\{1-\frac{3}{A}v'(D_{12}^B(P^B,P^B))\}}{(\{1-\frac{2}{A}v'(D_{11}^B(P^B,P^B))\}\{1-\frac{2}{A}v'(D_{12}^B(P^B,P^B))\}-\frac{v'(D_{11}^B(P^B,P^B))v'(D_{12}^B(P^B,P^B))}{A^2}}<0. \tag{56}$$

The preceding inequality follows Assumption 1. Moreover, based on Lemma 1(i), $\{-1/(\partial D_{11}^B(P^B, P^B)/\partial p_1^B)\}>0$, which implies that the first term in the bracket is negative.

Now, consider the second brackets. By summing Equations (40) and (41), we obtain

$$\frac{\partial D_{11}^B(p_1^B, p_2^B)}{\partial p_1^B}+\frac{\partial D_{11}^B(p_1^B, p_2^B)}{\partial p_2^B}$$
$$=\frac{-\frac{1}{A}+\frac{3}{A^2}v'(D_{12}^B(p_2^B, p_1^B))}{\{1-\frac{2}{A}v'(D_{11}^B(p_1^B,p_2^B))\}\{1-\frac{2}{A}v'(D_{12}^B(p_2^B, p_1^B))\}-\frac{v'(D_{11}^B(p_1^B, p_2^B))v'(D_{12}^B(p_2^B,p_1^B))}{A^2}}. \tag{57}$$

Differentiating Equation (57) with respect to p_1^B and evaluating the resulting equation at $p_1^B=p_2^B=P^B$, we obtain

$$\frac{\partial^2 D_{11}^B(P^B, P^B)}{\partial p_1^{B2}}\ \frac{\partial^2 D_{11}^B(P^B, P^B)}{\partial p_1^B\,\partial p_2^B}$$
$$=\frac{(H_D(p_1^B, p_2^B)\cdot\{\frac{3}{A^2}v''(D_{12}^B(P^B, P^B))\frac{\partial D_{12}^B(P^B, P^B))}{\partial p_1^B}\}}{H_D^2}$$
$$+\frac{1}{H_D(p_1^B, p_2^B)\}^2}\cdot[\frac{1}{A}\{1-\frac{3}{A}v'(D_{12}^B(P^B, P^B))\}]\cdot\frac{\partial H_D(P^B, P^B)}{\partial p_1^B}, \tag{58}$$

where

$$H_D(p_1^B, p_2^B)\equiv\{1-\frac{2}{A}v'(D_{11}^B(p_1^B, p_2^B))\}\{1-\frac{2}{A}v'(D_{12}^B(p_2^B, p_1^B))\}$$
$$-\frac{v'(D_{11}^B(p_1^B, p_2^B))v'(D_{12}^B(p_2^B, p_1^B))}{A^2}>0.$$

Given that $v''\leq 0$ and $(D_{12}^B)/(\partial p_1^B)>0$ based on Lemma 1(ii), the first term on the right side of Equation (58) is nonpositive.

For the second term on the right side of the equation, note first that $1/(\{H_D(p_1^B, p_2^B)\}^2)\cdot[(1/A)\{1-(3/A)v'(D_{12}^B(P^B, P^B))\}]>0$ based on Assumption 1. Moreover, we can demonstrate that $(\partial H_D(P^B, P^B))/(\partial p_1^B)<0$ as follows:

$$\frac{\partial H_D(P^B, P^B)}{\partial p_1^B}$$

$$=\frac{\partial}{\partial p_1^B}(1-\frac{2}{A}\{v'(D_{11}^B(p_1^B, p_2^B))+v'(D_{12}^B(p_2^B, p_1^B))\}$$

$$+\frac{3}{A^2}v'(D_{11}^B(p_1^B, p_2^B))v'(D_{12}^B(p_2^B, p_1^B)))\,|_{p_1^B=p_2^B=P^B}$$

$$=-\frac{2}{A}\{v''(D_{11}^B)\frac{\partial D_{11}^B}{\partial p_1^B}+v''(D_{12}^B)\frac{\partial D_{12}^B}{\partial p_1^B}\}+\frac{3}{A^2}\{v''(D_{11}^B)\frac{\partial D_{11}^B}{\partial p_1^B}v'(D_{12}^B)$$

$$+v''(D_{12}^B)\frac{\partial D_{12}^B}{\partial p_1^B}v'(D_{11}^B)\}$$

$$=-\frac{2}{A}v''(D_{11}^B)\cdot(\frac{\partial D_{11}^B}{\partial p_1^B}+\frac{\partial D_{12}^B}{\partial p_1^B})+\frac{3}{A^2}v''(D_{11}^B)v'(D_{11}^B)\cdot(\frac{\partial D_{11}^B}{\partial p_1^B}+\frac{\partial D_{12}^B}{\partial p_1^B})$$

$$=-v''(D_{11}^B)\cdot(\frac{\partial D_{11}^B}{\partial p_1^B}+\frac{\partial D_{12}^B}{\partial p_1^B})[\frac{1}{A}\{2-\frac{3}{A}v'(D_{11}^B)\}],$$

where the third equality applies the fact that under symmetric prices $p_1^B=p_2^B=P^B$, $v'(D_{11}^B)=v'(D_{12}^B)$ and $v''(D_{11}^B)=v''(D_{12}^B)$. Recall from Equation (56) that $(\partial D_{11}^B)/(\partial p_1^B)+(\partial D_{12}^B)/(\partial p_1^B)=(\partial D_{11}^B)/(\partial p_1^B)+(\partial D_{11}^B)/(\partial p_2^B)<0$. According to Assumption 1, $(1/A)\{2-(3/A)v'(D_{11}^B)\}>0$. Moreover, $v''\leq 0$. Therefore, we conclude that $(\partial H_D(P^B, P^B))/(\partial p_1^B)\leq 0$. In addition, the second term on the right side of Equation (58) is also nonpositive.

Given that both the first and the second terms on the right side of Equation (58) are nonpositive, $(\partial^2 D_{11}^B(P^B, P^B))/(\partial p_1^{B2})+(\partial^2 D_{11}^B(P^B, P^B))/(\partial p_1^B\partial p_2^B)\leq 0$. When this result is combined with $D_{11}^B(P^B, P^B)\cdot\{(\partial D_{11}^B(P^B, P^B))/(\partial p_1^B)\}^{-2}>0$, we conclude that the term in the second brackets in Equation (54) is nonpositive.

Given that the term in the first brackets in Equation (54) is negative and that in the second brackets is nonpositive,

$$\frac{d}{dP^B}(D_{11}(P^B, P^B)\cdot\{-\frac{1}{\frac{\partial D_{11}^B(P^B, P^B)}{\partial p_1^B}}\})<0.$$

The proof of Claim 1 is thus completed. □

F. Appendix 6

***Theorem* 3.**

Proof. Without losing generality, fix $i=1$. In equilibrium, the buyer-side and seller-side demands of platform 1 depend only on prices of this platform because the market for platform 1 is separated from that of platform 2. However, given that platforms 1 and 2 are symmetric, the equilibrium prices chosen by the two platforms agree with one another. Thus, subscript i is simply dropped in the prices, which are denoted as P^{B^*}, P^{S^*}, and $P^* \equiv P^{B^*}+P^{S^*}$.

The threshold $\bar{x}_{M1}$ in Figure 7 can be solved by solving

$$\{b^B+v(D_{11}^{B^*}(P^{B^*}))\}-P^{B^*}-t(\bar{x}_{M1}+\frac{\Delta}{2})=0,$$

which yields $\bar{x}_{M1}=(1/t)[\{b^B+v(D_{11}^{B^*}(P^{B^*}))\}-P^{B^*}-(\Delta/2)\,t]$.

Moreover, $\bar{x}_L$ is derived by solving

$$b_0^B-t(\bar{x}_L+\frac{\Delta}{2}+\delta)=\{b^B+v(D_{11}^{B^*}(P^{B^*}))\}-P^{B^*}-t(-\frac{\Delta}{2}-\bar{x}_L),$$

which yields $\bar{x}_L=(1/2t)[P^{B^*}-(b^B-b_0^B)-v(D_{11}^{B^*}(P^{B^*}))-t(\delta+\Delta)]$.

Thus, buyer-side demand for platform 1 is implicitly defined by

$$\begin{aligned} D_{11}^{B^*}(P^{B^*}) &= \frac{1}{2\delta+\Delta}(\bar{x}_{M1}-\bar{x}_L) \\ &= \frac{1}{A}[3\{b^B+v(D_{11}^B(P^{B^*}))-P^{B^*}\}+(-b_0^B+t\delta)]. \end{aligned}$$

Following a similar logic, seller-side demand for platform 1 is defined by

$$\begin{aligned} D_1^{S^*}(P^{S^*}) &= \frac{1}{(2\delta+\Delta)}(\bar{y}_{M1}-\bar{y}_L) \\ &= \frac{1}{A}[3(b^S-P^{S^*})+(-b_0^S+\tau\delta)]. \end{aligned}$$

The FOCs for the maximization problem of platform 1 are given by

$$P^{B^*}+P^{S^*}-c=\frac{1}{3}\{1-\frac{3v'(D_{11}^{B^*})}{A}\}[3\{b^B+v(D_{11}^{B^*})-P^{B^*}\}+(-b_0^B+t\delta)], \tag{59}$$

$$P^{B^*}+P^{S^*}-c=\frac{1}{3}[3(b^S-P^{S^*})+(-b_0^S+\tau\delta)]. \tag{60}$$

Equilibrium prices are denoted as $\hat{P}^{B^*}$, $\hat{P}^{S^*}$, and $\hat{P}^*\equiv\hat{P}^{B^*}+\hat{P}^{S^*}$.

As a benchmark, the case with no direct network effect (*i.e.*, $v(z)=0$, $\forall z\in[0, 1]$) is considered. In this case, the FOCs are given by

$$P^{B^*}+P^{S^*}-c=\frac{1}{3}[3(b^B-P^{B^*})+(-b_0^B+t\delta)], \tag{61}$$

$$P^{B^*}+P^{S^*}-c=\frac{1}{3}[3(b^S-P^{S^*})+(-b_0^S+\tau\delta)]. \tag{62}$$

The equilibrium prices in the benchmark case are denoted as $\tilde{P}^{B^*}$, $\tilde{P}^{S^*}$, and $\tilde{P}^*\equiv\tilde{P}^{B^*}+\tilde{P}^{S^*}$.

In both cases, $P^{B^*}+P^{S^*}-c=(1/3)[3(b^S-P^{S^*})+(-b_0^S+\tau\delta)]$ holds. By rearranging the terms, this equation is rewritten as $6P^{S^*}=3(b^S-P^{B^*}+c)+(-b_0^S+\tau\delta)$. Substituting this equation into Equations (59) and (61) yields

$$\begin{aligned}3(\hat{P}^{B^*}+b^S-c)+(-b_0^S+\tau\delta)=2\{1-\frac{3v'(\hat{D}_{11}^{B^*})}{A}\}\\ [3\{b^B+v(\hat{D}_{11}^{B^*})-\hat{P}^{B^*}\}+(-b_0^B+t\delta)],\end{aligned} \tag{63}$$

$$3(\tilde{P}^{B^*}+b^S-c)+(-b_0^S+\tau\delta)=2[3(b^B-\tilde{P}^{B^*})+(-b_0^B+t\delta)], \tag{64}$$

where Equations (63) and (64) are evaluated at the corresponding equilibrium prices, respectively.

Henceforth, the logic of the proof is exactly same as that of Theorem 1 and the proof will only be sketched. First, by using the definition of $D_{11}^{B^*}$, the right sides of Equations (63) and (64) can be rewritten as follows.

$$2\{1-\frac{3v'(D_{11}^{B^*})}{A}\}[3\{b^B+v(D_{11}^{B^*})-P^{B^*}\}+(-b_0^B+t\delta)]=2[A\cdot D_{11}^{B^*}-3v'(D_{11}^{B^*})D_{11}^{B^*}], \tag{65}$$

$$2[3(b^B-\tilde{P}^{B^*})+(-b_0^B+t\delta)]=2[A\cdot D_{11}^{B^*}-3v'(D_{11}^{B^*})]. \tag{66}$$

For a given price P^{B*}, $2[A\cdot D_{11}^{B*}-3v'(D_{11}^{B*})D_{11}^{B*}]\geq 2[A\cdot D_{11}^{B*}-3v'(D_{11}^{B*})]$ is equivalent to $(v(D_{11}^{B*}))/(D_{11}^{B*})\geq v'(D_{11}^{B*})$, which always holds via the concavity of $v(\cdot)$. Thus, we can conclude that

$$2\{1-\frac{3v'(\tilde{D}_{11}^{B*})}{A}\}[3\{b^B+v(\tilde{D}_{11}^{B*})-\tilde{P}^{B*}\}+(-b_0^B+t\delta)]>2[3(b^B-\tilde{P}^{B*})+(-b_0^B+t\delta)].$$

Second, given that $3(P^{B*}+b^S-c)+(-b_0^S+\tau\delta)$ is an increasing function of P^{B*}, the proof is completed by showing that

$$\frac{d}{dP^{B*}}(2\{1-\frac{3v'(D_{11}^{B*})}{A}\}[3\{b^B+v(D_{11}^{B*})-P^{B*}\}+(-b_0^B+t\delta)])<0,$$

which is confirmed true. □

G. Appendix 7

Theorem 4.

Proof. By using Equation (49) in Appendix 5, the closed form equilibrium prices of the model in Section IV can be calculated as follows:

$$\hat{P}^B=\frac{\frac{A-3\eta}{2A-3\eta}}{\frac{1}{3}+\frac{A-3\eta}{2A-3\eta}}\{(b^B-b_0^B)+t(\delta+\Delta)\}+\frac{\frac{1}{3}}{\frac{1}{3}+\frac{A-3\eta}{2A-3\eta}}(-1)\{(b^S-b_0^S)+\tau(\delta+\Delta)-c\},\tag{67}$$

$$\hat{P}^S=1/3[-2\hat{P}^B+(b^S-b_0^S)+2c+\tau(\delta+\Delta)].\tag{68}$$

Given that $(\partial/\partial\eta)((A-3\eta)/(2A-3\eta))=-3A/((2A-3\eta)^2)<0$, an increase in η reduces the weight of the first term in Equation (67) and increases that of the second term. This situation clearly reduces $\hat{P}^B$, and thus, increases $\hat{P}^S$. Using the same argument in the proof of Theorem 2, the overall price $\hat{P}$ decreases. □

H. Appendix 8

Theorem 5.

Proof. Differentiating Equation (67) with respect to τ yields $(\partial\hat{P}^B)/(\partial\tau)=-(1/3)/(1/3+(A-3\eta)/(2A-3\eta))(\delta+\Delta)$. For any $\eta>0$ that satisfies

Assumption 1, this value is smaller than that of the benchmark case ($\eta=0$)$(\partial\tilde{P}^B)/\partial\tau=-2/5(\delta+\Delta)$, which is smaller than 0. Thus, for any given $\eta>0$, an increase in τ reduces $\hat{P}^B$ more severely than in the case $\eta=0$, which implies that the variations of $\hat{P}^S$ and $\hat{P}$ are also larger under direct network effect. □

I. Appendix 9

***Theorem* 6.**

Proof. $A\equiv(2\delta+\Delta)2t$, and thus, $(\partial/\partial t)((A-3\eta)/(2A-3\eta))=((2\delta+\Delta)6\eta)/((2A-3\eta)^2)>0$. Therefore, increments in t increases both the weight of the first term in Equation (67) and the first term itself, which results in higher $\hat{P}^B$. This situation clearly reduces $\hat{P}^S$ and increases $\hat{P}$.

Denote $G\equiv(A-3\eta)/(2A-3\eta)$ and $H\equiv(\partial/\partial t)((A-3\eta)/(2A-3\eta))=((2\delta+\Delta)6\eta)/((2A-3\eta)^2)(>0)$. By rearranging Equation (67), the following is obtained:

$$\{\frac{1}{3}+G\}\hat{P}^B=G\{(b^B-b_0^B)+t(\delta+\Delta)\}+\frac{1}{3}(-1)\{(b^S-b_0^S)+\tau(\delta+\Delta)-c\}.$$

Differentiating the preceding equation with respect to t and rearranging its terms yield

$$\begin{aligned}\frac{\partial\hat{P}^B}{\partial t}&=\frac{H}{1/3+G}\{(b^B-b_0^B)+t(\delta+\Delta)-\hat{P}^B\}+\frac{G}{1/3+G}(\delta+\Delta)\\&=\frac{H}{1/3+G}[(\frac{1}{1+3G})\{(b^B-b_0^B)+t(\delta+\Delta)+(b^S-b_0^S)+\tau(\delta+\Delta)-c\}]\\&+\frac{G}{1/3+G}(\delta+\Delta).\end{aligned}\tag{69}$$

Note that $(\partial\tilde{P}^B)/\partial t=(3/5)(\delta+\Delta)$. The last term in Equation (69) is evidently smaller than $(\partial\tilde{P}^B)/\partial t=(3/5)(\delta+\Delta)$ if $\eta>0$. However, the first term in Equation (69) is positive, and thus, the relative sizes of $(\partial\hat{P}^B)/\partial t$ and $(\partial\tilde{P}^B)/\partial t$ remain ambiguous. □

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